Journées des jeunes en cotutelle

Abstracts of talks

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Jean-Yves Fortin

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Boundary field induced first order transition in the 2D Ising model: Exact study using Grassmann algebra

We present in this seminar an exact study concerning a first order transition induced by an inhomogeneous boundary magnetic field in the 2D Ising model. From the analysis of the interfacial free energy in the discrete case using Grassmann techniques, we identify, by the mean of an asymptotic expansion in the thermodynamic limit, the line of transition that separates the regime where the interface is localised near the boundary from the one where it is propagating inside the bulk. We also show how this critical line is dependent on the aspect ratio of the lattice.

Alexey Glutsyuk

Ecole Normale Supérieure de Lyon

On noncommutative rational approximations

It is well known that the best approximation of an irrational number by rationals is given by the successive ratios of its continued fraction: the corresponding approximation rate is estimated by a power (arbitrarily close to the inverse square) of the denominator. We will discuss some noncommutative versions.

Question 1. Given a (semisimple) Lie group G (e.g., $SL_2(\mathbb{R})$ or $SO(3)$) and a pair of elements $(A, B) \in G \times G$ generating a dense subgroup. What is the best rate of approximation of an element of G by words in A and B (in terms of the lengths of the words)?

It is well-known (an old result of D.Epstein) that almost each pair of elements (A, B) in a nonsolvable Lie group G generates a free subgroup. On the other hand, any pair (A, B) generating a nondiscrete (free) subgroup can be approximated by a sequence $(A_n, B_n) \to (A, B)$ of pairs generating nonfree subgroups. This was proved by the speaker.

Question 2. Estimate the best approximation rate in terms of the minimal length of relation in the approximating nonfree group.

These Questions are closely related to each other. They are widely open, even for the groups $SL_2(\mathbb{R})$ and $SO(3)$. There is a conjecture saying that generically the best approximation rate should be exponential in the lengths of the words (relations).

We will give a survey of related results.

Sergey Gorchinskiy

Steklov Mathematical Institute, Moscow

Degree of the modular map of maximal variation families of n-gonal curves (joint work with Filippo Viviani)

The question in subject is the study of numerical properties of families of certain algebraic curves with certain maximal properties. More precisely, we consider the degree of the modular map, i.e. a map to the moduli space of the corresponding type curves.

First, hyperelliptic curves are discussed. Namely, we study differences between the coarse moduli space (whose points correspond to hyperelliptic cuvres) and the stack of hyperelliptic curves (which represents the functor of families of hyperelliptic curves). In particular, we compute their Picard groups, giving explicit description of the generators. We also study how many families of hyperelliptic curves may have the same modular map as well as the existence of the tautological family over the coarse moduli space of hyperelliptic curves. Finally, we get an application to a possible degree of modular map.

Next, we study trigonal curves, i.e. curves, which admit a degree three map to a projective line. Certain numerical results on the degree of the modular map are obtained for families of such curves. In particular, the description of the relative Picard group of the universal family of trigonal curves is used.

Finally, we state some conjectures about n-gonal curves, providing a general construction of embedding into a rational normall scroll, which reduces these conjectures to some geometrical questions about scrolls.

Anton Khoroshkin

Institut des Hautes Etudes Scientifiques, Bures-sur-Yvette; Independent University of Moscow

The character of the operad of a pair of compatible brackets

A pair of Poisson brackets is called compatible if any their linear combination is also a Poisson bracket. The space of all compositions of this skew-commutative operations generates a quadratic operad. We describe of the action of n-th symmetric group on the operations with n inputs generalising the Kljachko's theorem. The proof of this theorem is based on the Koszul duality for operads.

Victor Kleptsyn

Université de Genève

Lyapunov exponents of transversally conformal foliations (joint work with B. Deroin)

Given a foliation with a Riemannian metrics on its leaves, one can consider the corresponding leafwise Brownian motion. Study of such a motion, its stationary measure and its characterisation in terms of the leafwise Laplace operator was begun by a work of Garnett, later followed by many other authors.

We study the behavior of the holonomy maps along typical Brownian trajectories, under an additional assumption that the foliation is transversally conformal (for instance, of real or complex codimension one). Our main result is: either there exists a transversely invariant measure (a rather rare case), or for a typical Brownian path the corresponding holonomy maps contract some transversal neighborhood exponentially. To obtain this result, we study the Lyapunov exponent of the foliation; negative Lyapunov exponent signifies exponential contraction, and in the case of absence of a negative Lyapunov exponent a transversely invariant measure is constructed. As a corollary of this result, in the case of absence of a transversely invariant measure we obtain the uniqueness of a harmonic measure on every minimal subset of our foliation.

The "Sullivan's dictionary" allows us to extend these results to the other cases of conformal dynamical systems, that is, conformal group and pseudo-group actions and conformal correspondences.

Grégoire Lecerf

CNRS; Laboratoire J.-V. Poncelet, Moscow

A Gröbner free alternative for polynomial system solving

In this talk we will present a recent probabilistic algorithm for solving systems of polynomial equations and inequations. Our algorithm computes the equidimensional decomposition of the Zariski closure of the solution set of such systems. Each equidimensional component is encoded by a generic fiber, that is a finite set of points obtained from the intersection of the component with a generic transverse affine subspace. Our algorithm is incremental in the number of equations to be solved. Its cost is mainly cubic in the maximum of the degrees of the solution sets of the intermediate systems counting multiplicities. This behavior is reached thanks to certain lifting methods that generalize the classical Newton operator.

Yury Lifshits

Steklov Mathematical Institute, Saint-Petersburg branch

Algorithms for processing automatically generated texts

How to solve string problems, if instead of input string we get only program generating it? Is it possible to solve problems faster than just "generate text + apply classical algorithm"?

In this talk we consider strings generated by straight-line programs (SLP). These are programs using only assignment operator. We show new algorithms for equivalence, pattern matching, finding periods and covers, computing fingerprint table on SLP-generated strings. ¿From the other hand, computing the Hamming distance and the longest common subsequence is NP-hard.

Main corollary is an $O(n^{2m})$ algorithm for pattern matching in LZ-compressed texts.

Denis Mironov

Steklov Mathematical Institute, Moscow; Laboratoire J.-V. Poncelet, Moscow

Equivariant embeddings of homogeneous spaces of reductive groups into G-modules

The topic of this talk revolves around embeddings of homogeneous spaces of reductive groups into their simple or semisimple representations. The main case which will be considered studies homogeneous space of reductive subgroup and its embeddings into simple G-modules. Necessary and sufficient conditions will be given. It is proven, that obstacles lie in character group of $Aut_G(G/H)$ for spherical homogeneous spaces. More general case gives relation between action of $Aut_G(G/H)$ on simple G-module and possibility of equivariant embedding $G/H \to V$. Also this method can be extended to embeddings into direct sums of simple G-modules. In addition to this we will briefly consider the case of unipotent radical of parabolic subgroups.

Mathieu Raffinot

CNRS; Laboratoire J.-V. Poncelet, Moscow

Computing common intervals of K permutations, with applications to modular decomposition of graphs

We introduce a new approach to compute common intervals of K permutations based on a very simple and general notion of generators of common intervals. This formalism leads to simple and efficient algorithms to compute the set of all common intervals of K permutations, that can contain a quadratic number of intervals, as well as a linear space basis of this set of common intervals. Finally, we show how our results on permutations can be used for computing the modular decomposition of graphs in linear time.

Sergey Rybakov

Laboratoire J.-V. Poncelet, Moscow

Drinfeld modules and coding theory

We will start with a question from coding theory: how to construct a curve over $\mathbb{F}_q$ of given genus g with many points, and an assimptotic analogue of this question when g tends to infinity. In attempt to give an answer we will eventually come to Drinfeld modules and their moduli schemes. I will explain basic definitions and results on Drinfeld modules and how to apply Drinfeld moduli schemes to the problem we started with.

Leonid Rybnikov

Laboratoire J.-V. Poncelet, Moscow

Shift of invariants

Shift of invariants is a powerful method to construct commutative subalgebras in Poisson algebras. For any semisimple Lie algebra $\mathfrak{g}$ this method gives us a family of maximal Poisson commutative subalgebras in the symmetric algebra $S(\mathfrak{g})$. I will discuss the "quantum" analogues of these subalgebras. They form a family of maximal commutative subalgebras in the universal enveloping algebra $U(\mathfrak{g})$. The extreme points of this family turn out to be related to the Gelfand–Tsetlin bases. On the other hand, this family of commutative subalgebras is related to the Gaudin model and Bethe-Ansatz.

Evgeny Smirnov

Institut Fourier, Grenoble; Laboratoire J.-V. Poncelet, Moscow

Bruhat decomposition for double Grassmannians

The classical Schubert calculus describes the combinatorics and geometry of orbit closures of a Borel subgroup $B \subset \mathrm{GL}(n)$ acting on a Grassmann variety $\mathrm{Gr}(k, n)$. The same question could be considered for the direct product of two Grassmannians $\mathrm{Gr}(k, n) \times \mathrm{Gr}(l, n)$ and the diagonal action of the subgroup of upper-triangular matrices $B \subset \mathrm{GL}(n)$ on this product. This is equivalent to classifying configurations of triples (k-plane, l-plane, full flag) in an n-dimensional vector space up to the $\mathrm{GL}(n)$-action. It turns out that the number of such configuration is finite.

In my talk I will present a combinatorial description of these B-orbits and construct desingularizations of their closures analogous to the Bott–Samelson desingularizations for Schubert varieties. I am also going to mention how are these varieties related to representations of quivers.

// Alexey Zykin

Laboratoire J.-V. Poncelet, Moscow

On the generalized Brauer–Siegel theorem

The classical Brauer-Siegel theorem states that if k runs through the sequence of normal extensions of $\mathbb{Q}$ such that $n_k/\log|D_k| \to 0$, then

$$\log h_k R_k / \log \sqrt{|D_k|} \to 1.$$

In this talk we will give a brief survey of what is known about the generalizations of this theorem. First, we will discuss the versions of the Brauer-Siegel theorem where the conditions on the family of number fields are considerably weakened. Second, we will dwell on the higher dimensional analogues of the Brauer-Siegel theorem both in the number field and in the function field cases giving an overview of known results and open problems.