School
"Flows on surfaces, symbolic dynamics and dynamics in moduli spaces."

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ABSTRACTS

Ergodic integrals for translation flows
Alexander Bufetov

In 1982 H.Masur and W.Veech proved that a generic translation flow is uniquely ergodic. This raises a natural question about the speed of convergence in the ergodic theorem for these systems. Logarithmic estimates on the speed of convergence were given by A.Zorich and G.Forni. G.Forni classified the space of distributions (in the sense of Sobolev and Schwartz) invariant under translation flows. An asymptotic formula for ergodic integrals can be given in terms of special finitely-additive measures invariant under our dynamical system. In particular these finitely-additive measures give an explicit construction of G.Forni's invariant distributions. The minicours will give an elementary detailed introduction to the subject.

Decays of the cocycle and of the coefficients of the quasi-regular representation of $SL(2,\mathbb{R})$
Christophe Pittet

The cocycle of the quasi-regular representation of a semi-simple Lie group G on its Furstenberg boundary is related to the Harish-Chandra function of G. The Harish-Chandra function is a diagonal coefficient which is stable. Other coefficients are unstable. We will give a geometric explanation of this stable/unstable properties and ask for a better understanding in dynamical terms. We will focus on the simplest case $G=SL(2,\mathbb{R})$.

Products of random matrices
Jean-Francois Quint

I will give a detailed presentation of the Furstenberg-Kesten theory of non-degenerate product of random matrices in $SL(2,\mathbb{R})$. We will get positivity of the first Lyapunov exponent and uniqueness of the stationary probability measure on the projective line.

Basics in ergodic theory
Jean-Francois Quint

I will review general notions of ergodic theory and give practical examples in homogeneous dynamics.

Very flat surfaces and dynamics in the moduli spaces.
Anton Zorich

Certain problems of billiards in polygons, of measured foliations on surfaces, of interval exchange transformations, and some others have a reformulation in terms of geometry of the associated surface endowed with a flat metric with trivial holonomy and with several conical singularities. Such a metric is, basically, the same as a holomorphic 1-form on a Riemann surface. The geometry of an individual flat surface in many aspects depends on the behavior of the Teichmüller geodesic passing through the associated point of the moduli space. Recently Alex Eskin and Maryam Mirzakhani have proved that the closure of any complex Teichmüller geodesic is a very sympathetic orbifold.

In my lectures I will try to tell about key geometric structures involved in this story, and about relations between them.