Ballistic deposition model NNN case

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$$h(i,t+1) = \max[h(i-1,t),\, h(i,t),\, h(i+1,t)] + 1$$

$$[\mathrm{Var}\, h(i,t)]^{1/2} = \frac{1}{N^{1/2}} \left[\sum_{i=1}^{N} h^2(i,t) - h(t)^2 \right]^{1/2} = N^{1/2} g\left(\tau/N^{3/2}\right)$$

the variable $\tau = T/N$ is the averaged number of particles per one column, and the function $g(u)$ for the rescaled variable $u = \tau/N^{3/2}$ has the following asymptotic behavior: $g(u) \sim u^{1/3}$ for $u \ll 1$ and $g(u) \sim \text{const}$ for $u \gg 1$.

$$
g_i = \begin{pmatrix} 1 & & & & \\ & \ddots & & & \\ & & \boxed{\begin{matrix} 1 & 0 & 0 \\ u & u & u \\ 0 & 0 & 1 \end{matrix}} & & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix} \leftarrow \text{row } i
$$

Elementary act of the deposition in the matrix form

$$V(N,u) =: \prod_{t=1}^{T} g_{i_t}:$$

It is the description of the whole heap

Each element of the matrix $V(N,u)$ is a polynomial of the variable u. Take a vector $\mathbf{a}(t=0) = (a_1, ..., a_N)$ where $a_i (i = 1, ..., N)$ are distinct nonzero values. The set of local heights $\mathbf{h}(T) = (h_1(T), ..., h_N(T))$ at time t after deposition event can be extracted as follows:

$$\mathbf{h}(T) = \lim_{u \to \infty} \frac{\ln[\hat{V}(T,u)\,\mathbf{a}(t=0)]}{\ln u} \tag{2.6}$$

$$\begin{pmatrix} a_1(t+1) \\ \vdots \\ \\ a_{j-1}(t+1) \\ a_j(t+1) \\ a_{j+1}(t+1) \\ \\ \vdots \\ a_N(t+1) \end{pmatrix} = \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & \boxed{\begin{matrix} 1 & 0 & 0 \\ u & u & u \\ 0 & 0 & 1 \end{matrix}} & \\ & & & \ddots \\ & & & & 1 \end{pmatrix} \begin{pmatrix} a_1(t) \\ \vdots \\ \\ a_{j-1}(t) \\ a_j(t) \\ a_{j+1}(t) \\ \\ \vdots \\ a_N(t) \end{pmatrix}$$

If $i_t = i$ (i.e. we drop at particle in the column i ar time t), then

$$a(i, t+1) = u\, a(i-1, t) + u\, a(i, t) + u\, a(i+1, t)$$

Let us consider the following anzatz

$$a(i,t) = u^{h(i,t)}$$

We get the following equation

$$h(i,t+1) = \frac{1}{\ln u} \ln \left[e^{(h(i-1,t)+1)\ln u} + e^{(h(i,t)+1)\ln u} + e^{(h(i+1,t)+1)\ln u} \right]$$

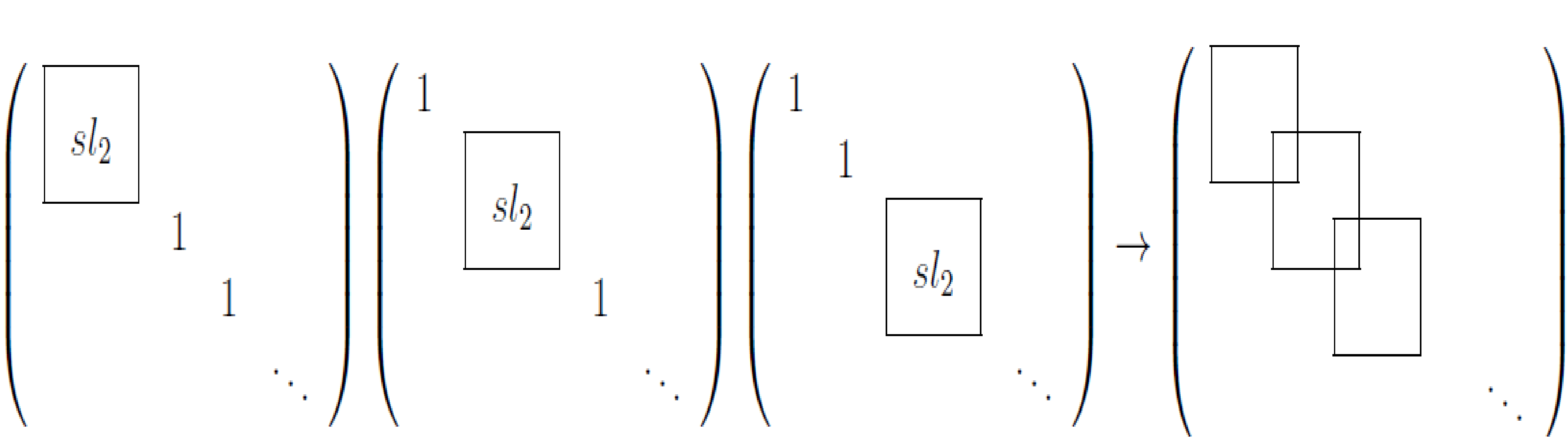
The BD process as Markov process on the local semigroup with nontrivial local relations

$$g_k g_m = g_m g_k \qquad \forall |k-m| \geq 2, \qquad \{k,m\} = 1,...,N$$

Each pair of neighboring generators $g_k, g_{k\pm1}$ produces a free subgroup (sub–semigroup) of a group F (semi–group F^+).

$$g_i(t) = \begin{pmatrix} 1 & & & & \\ & \ddots & & & \\ & & \boxed{U_i^{(t)}} & & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix}; \qquad U_i^{(t)} = \begin{pmatrix} a_i^{(t)} & b_i^{(t)} \\ c_i^{(t)} & d_i^{(t)} \end{pmatrix}; \qquad a_i^{(t)} d_i^{(t)} - b_i^{(t)} c_i^{(t)} = 1$$

Schematic realization of the BD problem

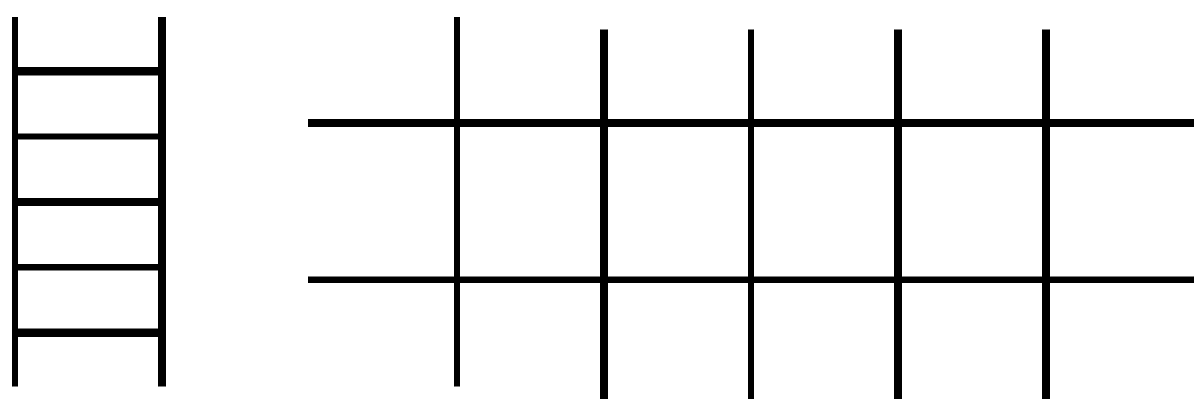


Diffusion equation reduces to the Toda system

$$\partial_t U(\mu_1, \mu_2, t) = D \left[\frac{1}{3} \left(\partial^2_{\mu_1} + \partial^2_{\mu_2} + \partial_{\mu_1} \partial_{\mu_2} \right) + e^{\mu_1 - 2\mu_2} f + e^{-2\mu_1 + \mu_2} g \right] U(\mu_1, \mu_2, t)$$

The system asquires the form of the Toda chain in the rank 2 matrix formulation. However there is equivalent rank N Toda formulation which corresponds to the free motion on the symmetric space.

Geometrically it corresponds to the rotation of the «brane configuration»



The disordered wires system(Calogero model)

$$W(\mu_1, ..., \mu_N), t) \ = \ \xi(\mu_1, ..., \mu_N)\Psi(\mu_1, ..., \mu_N), t)$$

where

$$\xi\{\mathbf{x}\} = \prod_{\alpha \in R_+} (\sinh x_\alpha)^{\nu_\alpha} = \prod_{i<j} \left|\sinh^2 x_j - \sinh^2 x_i\right|^{\beta/2} \prod_i |\sinh 2x_i|^{1/2}$$

$$-\frac{\partial}{\partial t}\Psi(\mu, t) = H\Psi(\mu, t)$$

$$H = -\frac{1}{2}\sum_j \left(\frac{\partial^2}{\partial x_j^2} + \frac{1}{\sinh^2 2x_j}\right) + \frac{\beta(\beta-2)}{4}\sum_{j<k}\left(\frac{1}{\sinh^2(x_k - x_j)} + \frac{1}{\sinh^2(x_k + x_j)}\right) + c$$

There is so called Inozemtsev limit when hyperbolic Calogero system gets reduced to the Toda chain

$$\begin{cases} \beta = -\sqrt{2f}e^{\Delta} \\ x_j = \mu_j + j\Delta \end{cases} \qquad (\Delta \to \infty)$$

The corresponding limit in the statistical models corresponds to the relation betweem the transmission in the disordered wires (Calogero model) and ballistic deposition problem with SL(2,R) uniform multi-column box