CURRICULUM VITAE

VICTOR A. VASSILIEV

MATHEMATICS COLLEGE — INDEPENDENT UNIVERSITY OF MOSCOW
STEKLOV MATHEMATICAL INSTITUTE — 8 GUBKINA ST. 117966
MOSCOW, RUSSIA
Telephone: [7-095]-151-2629; -515-6266
e-mail: `vassil@vassil.mccme.rssi.ru`

PERSONAL INFORMATION

Born: April 10, 1956 **Citizenship:** citizen of Russia
Married: to Tatiana Vassilieva **Children:** Maria (1982), Fedor (1995), Ivan (1996).

EDUCATION

Dr. of Sci., 1992	Mathematics, Steklov Mathematical Institute of Russian Ac. Sci., Moscow	
Ph.D., 1982	Mathematics, Moscow State University (MGU)	Advisor: V. I. Arnold
M.S., 1978	Mathematics, Moscow State University (MGU)	

AREA OF SPECIALIZATION

Singularity Theory; Topology;
Complexity Theory; Integral Geometry;
Symplectic Geometry and Topology; Partial Differential Equations;
Complex Analysis; Combinatorics.

PROFESSIONAL EXPERIENCE

1991 – now	Professor, Mathematics College, Independent Moscow University
1995 – 1997	Leading researcher,
1997 – now	Principal researcher, Department of Geometry and Topology, Steklov Mathematical Institute of Russian Ac. Sci., Moscow
1990 – 1995	Leading researcher, Department of Mathematics, Research Institute for System Studies, Moscow
1989 – 90	Senior researcher, Department of Functional Analysis, Institute of Applied Mathematics, Russian Ac. of Sci., Moscow
1987 – 89	Senior researcher, Statistical Information Systems Research Institute, Moscow
1982 – 86	Teacher, Specialized Mathematical High School # 57, Moscow (part-time)
1981 – 87	Senior researcher, Documents and Archives Research Institute, Moscow

GRANTS AND AWARDS

- Moscow Mathematical Society Award, 1986.
- MSRI Research Professorship, 1997.

- Miller Professorship, Berkeley Univ., 1999.
- Visiting Fellow Commonership, Trinity College, Cambridge University, 2000.
- AMS fSU Aid Fund grant, 1993.
- International Science Foundation Research grant, 1994 – 95. (head of a research team).
- INTAS grant, 1995 – 96 (head of a research team).
- Research grant of the Russian Fund of Basic Research (RFBR), 1995–1997 (head of a research team).
- Research grant of the Netherlands Organization for Scientific Research (NWO), 1996.
- INTAS grant, 1997 – 98 (head of a research team).
- RFBR grant for support of distinguished scientific schools, 1997–99 (co-head).
- Research grant of RFBR, 1998–2000 (head of a research team).
- RFBR grant for support of distinguished scientific schools, 2000–2002 (co-head).
- Research grant of the Netherlands Organization for Scientific Research (NWO), 2000–2002.
- Research grant of RFBR, 2001–2003 (head of a research team).
- INTAS grant, 2001 – 2002 (head of a research team).
- Russian President's grant for support of distinguished scientific schools, 2003–2005 (co-head).

PROFESSIONAL ACTIVITIES AND SERVICES

- Chairman of the Commission of Russian Ac. Sci. on teaching mathematics in schools (since 2004, member since 2001)
- Member at large, Executive Committee of the International Mathematical Union, 2004–2006
- Russian Academy of Sciences, ordinary member (since 2003, corresponding member since 1997)
- Moscow Mathematical Society, member (since 1984), board member (since 1992), vice-president (since 2004)
- American Mathematical Society, member (since 1995)
- Board member of the Independent University of Moscow (since 2004)

- Periodical edition "Sovremennye problemy matematiki" (Contemporary problems of Mathematics) VINITI, chief editor (since 2004)
- Journal "Functional Analysis and its Applications", vice-chief editor (since 1995)
- Journals "Selecta Mathematica. New Series", "Journal of Knot Theory and its Ramifications", "Moscow Mathematical Journal", "Topology and its Applications", "Izvestiya of Russian Ac. Sci., Ser. Math." editorial board member (since 1994, 1992, 2000, 2003 and 2003 respectively)
- MIR Publishers, Scientific editor of Russian translations of Proceedings of Bourbaki Seminar (1991–96)
- Federal Expert Council for selection of recommended textbooks in mathematics, member (2004–2005)
- Member of the Organizing Committee of the 13 All-Union School on the Operator Theory. Kuybyshev, October 1988.
- Member of the Organizing Committee of the International Geometrical Colloquium. Moscow, May 1993.
- Member of the Organizing Committee of the International Conference dedicated to 60-th anniversary of V.I. Arnold (Toronto, 1997)
- Member of the Program Committee of the L.S. Pontryagin memorial conference, Moscow August–September 1998. (Head of the Topology Section.)
- Co-organizer of a conference on Singularity Theory, Oberwolfach, May 1999.
- Member of the Organizing Committee of the NATO Advanced Study Institute on Singularity Theory at the Newton Mathematical Institute, Cambridge, UK, July–August 2000
- Chairman of the Organizing Committee of the Moscow Mathematical Olympics, 2001
- Co-director of the NATO Advanced Research Workshop on New Techniques in Topological Quantum Field Theory, Calgary, Canada, August 2001
- Co-organizer of a conference on Singularity Theory, Oberwolfach, September 2001.
- Member of the program committee of the international conference "Fundamental Mathematics Today" dedicated to the 10th anniversary of the Independent Moscow University, Moscow, December 26–29, 2001
- Member of the organizing committee of the International conference "Singularity Theory and Bifurcations", Steklov Math. Inst., Moscow, August 2002.
- Co-organizer of a conference on Singularity Theory, Oberwolfach, September 2003.
- Member of the program committee of the Petrovsky memorial conference, Moscow 2004

- Chairman of the organizing committee of the summer mathematical school (Dubna, 2004)
- Chairman of the organizing committee of the conference "Mathematical education in contemporary circumstances" of the Russian association of Math. teachers, Perm, 2004.
- Co-organizer of a conference on Singularity Theory, Oberwolfach, September 2006.
- Member of the Program Committee of the International Congress on Mathematical Education, Monterrey, Mexico, 2008.

VISITING POSITIONS

- Visiting Fellow Commoner, Trinity College, Cambridge UK, October-December 2000
- Invited Researcher, Univercité de Rennes I, April-May 2000
- Miller Visiting Research Professor, UC Berkeley, January-April 1999.
- Research Professor, MSRI, January-March 1997
- Univercité Denis Diderot (Paris-7), Professor, May 1994
- University of Maryland, Visiting Researcher, February-March 1993
- Univercité Denis Diderot (Paris-7), Associate Professor (Maitre des Conferences), June-July 1992

TEACHING EXPERIENCE

In 1973-81, being a student of the Moscow State University, I taught at the Mathematics Correspondence School created by I.M. Gel'fand for high school students from all parts of Russia.

In 1982-86 I taught mathematics at the specialized Moscow Mathematical High School # 57.

In 1987-89 I taught the special courses on Topology for advanced students of this school.

In 1987-91 I taught informal lecture courses on Topology and Singularity Theory for undergraduate students. Among my students in these years were A. Polishchuk, L. Possitselsky, A. Stoyanovsky, R. Bezrukavnikov, N. Nekrasov, M. Entov, S. Barannikov, P. Pushkar', Yu. Makhlin, T. Misirpashaev, O. Kravchenko, S. Shteingold, G. Vinner, M. Braverman and several other currently professional mathematicians and physicists.

In 1991-92 and 92-93 academic years I taught Calculus at the Moscow Independent University, in 1993-94, 1996, Fall 1998, 1999-2000 and 2001 lecture courses on Topology, in 1994-95 on Singularity Theory, in Fall 1995 on Topology of Singularities, in 1997-98 on Additional Chapters of Topology, and in 2002-2003 on Topology of Discriminants and Plane Arrangements at the same university. In 2001-2002 I taught Topology-2 at the MATH IN MOSCOW program at the same university.

Ph.D. Theses supervised: S.S. Anisov (1998), N.S. Markarian (1999), V.E. Turchin (2000), A.G. Gorinov (2004).

INVITED LECTURES AT INTERNATIONAL CONFERENCES

- **Cohomology of braid groups and complexity,** Conference "Unity and Diversity in Mathematical Science" (Smalefest), Berkeley, August 1990
- **Title forgotten, the first public presentation of finite type knot invariants;** Colloquium on quantum groups and knot theory, Leningrad, December 1990
- Conference "Differential equations and related problems", Moscow, May-June 1991
- **Complements to discriminants of smooth mappings,** College on singularity theory, Trieste, ICTP, August – September 1991
- **Knot invariants and singularity theory**, College on singularity theory, Trieste, ICTP, August – September 1991
- Rencontre Franco-Russe on Geometry, Luminy (France), May 1992.
- International conference dedicated to the memory of A.N. Kolmogorov, Euler Institute, St.-Petersburg, May-June 1993
- International Topology Conference, Athens (Georgia, USA), August 1993
- **Ramification in integral geometry and monodromy of complex links**, French–Russian conference, Moscow, RAS Inst. of Control Problems, 1993
- Conference on moduli spaces of curves, Texel Island, Netherlands, April 1994
- International colloquium dedicated to the 65-th anniversary of Jean Cerf, Bures-Sur-Yvette, May 1994
- **Topology of discriminants and their complements, International Congress of Mathematicians, Zürich, August 1994, Plenary (one-hour) lecture**
- Joint meeting of the I.G. Petrovskiĭ seminar and Moscow Mathematical Society, Moscow, January 1995
- **Topological complexity of root-finding algorithms,** AMS-SIAM seminar "The mathematics of numerical analysis", Park City (Utah, USA), July-August 1995
- **Algebraicity of surface potentials and monodromy of complete intersections,** Conference on Singularity Theory, Oberwolfach, July 1996
- **Topology of two-connected graphs and homology of spaces of knots,** Joint Workshop "Combinatorial Aspects of Finite Type and Quantum Invariants", MSRI, Berkeley, January 1997
- Workshop "Geometric Combinatorics", MSRI, Berkeley, February 1997

- **Homology of i-connected graphs and invariants of knots, plane arrangements, etc.** International conference dedicated to the 60-th anniversary of V.I. Arnold, Toronto, June 1997
- "Solitons, Geometry and Topology: on crossroads." International conference dedicated to the 60-th anniversary of S.P. Novikov, Moscow, May 1998.
- **Topological order complexes and resolutions of discriminant sets,** International conference on Geometric Combinatorics, Kotor, Montenegro, August-September 1998
- **Topological order complexes and topology of discriminants,** International conference dedicated to 90-th anniversary of L.S. Pontryagin, Moscow, August–September 1998.
- **Order complexes of singular sets and topology of spaces of nonsingular projective varieties,** Conference on Singularity Theory, Oberwolfach, May 1999
- **Order complexes of singular sets and topology of spaces of nonsingular projective varieties,** International conference dedicated to the 80-th anniversary of V.A. Rokhlin, St. Petersburg, August 1999.
- **Classification and enumeration of topologically distinct morsifications of real function singularities**, 6-th International Symposium on Effective Methods in Algebraic Geometry (MEGA), Bath (UK), June 2000
- **Resolutions of discriminants and topology of their complements,** course of two lectures, Advanced Study Institute at Newton Inst., Cambridge UK, July–August 2000.
- **Homology of spaces of knots in any dimensions,** Royal Society Discussion Meeting "Topological Methods in Physical Sciences", London, 15–16 November 2000
- **Eilenberg–Moore–Anderson spectral sequence as a banal application of the discriminant topology**, Informal Workshop in Singularity Theory, November 24–25 2000, Newton Mathematical Institute, Cambridge UK
- **Geometry of discriminants and computation of knot invariants**, Workshop in Singularity Theory, 17–19 May 2001, Utrecht, the Netherlands
- **Theory of lacunas and Petrovskii condition for hyperbolic operators**, I.G.Petrovsky Centennial Conference, Moscow, 22–27 May, 2001
- Conference on Monodromy Theory, Steklov Mathematical Institute, June 2001
- **Combinatorial formulas for cohomology of spaces of knots,** Research workshop in Low-Dimensional Topology, Calgary, August 2001
- **Combinatorial formulas for cohomology of spaces of knots,** International Conference "Fundamental Mathematics Today" dedicated to the 10th anniversary of the Independent University of Moscow, Moscow, December 2001

- **Geometry of Discriminants and Classical Homotopy Topology,** International Conference "Kolmogorov and contemporary mathematics", Moscow, June 16–21, 2003

- **On the cohomology of spaces of self-intersecting curves,** International Conference "Combinatorial Methods in Physics and Knot Theory", Laboratoire J.-V. Poncelet and Independent University of Moscow, 21–25 Febr. 2005.

PUBLICATIONS

Books

[1] **Lagrange and Legendre characteristic classes**. Gordon and Breach Publishers. New York a.o., 1988, 274 pp.

[2] Second edition of [1], 1993, 273 pp.

[3] Extended Russian translation of [1]; M., MCCME, 2000, 312 p.

[4] **Singularities. I. Local and global theory** (with V.I. Arnold, V.V. Goryunov and O.V. Lyashko). Moscow, VINITI, 1988, 256 pp. (in Russian).

[5] English translation of [4]. Encycl. Math. Sci., vol.6 (Dynamical systems VI), Springer–Verlag, Berlin & New York, 1993, 245 pp.

[6] Second edition of [5], 1998.

[7] **Singularities. II. Classification and applications** (with V.I. Arnold, V.V. Goryunov and O.V. Lyashko). Moscow, VINITI, 1989, 256 pp.

[8] English translation of [7]. Encycl. Math. Sci., vol. 39 (Dynamical systems VIII), Springer–Verlag, Berlin & New York, 1993, 233 pp.

[9] **Complements of discriminants of smooth maps: topology and applications.** Translations of Math. Monographs, AMS, Providence RI, 1992, 210 pp.

[10] Revised and enlarged edition of [9], 1994, 268 pp.

[11] **Ramified integrals, singularities and lacunas**. Kluwer Academic Publishers, Dordecht (Netherlands), 1994, 289+xvii pp.

[12] **Topology of complements of discriminants,** Moscow, Phasis, 1997, 552 pp. (in Russian).

[13] **Vetvyashiesya Integraly (Ramified Integrals)**, Moscow, MCCME, 2000 (in Russian), 432 pp.

[14] **Introduction to Topology,** Moscow, Phasis, 1997, 144 pp. (in Russian).

[15] English translation of [14], AMS, 2001, 149+xiii pp.

[16] Second revised edition of [14], Moscow, Phasis, 3000 (in Russian, to appear).

[17] **Applied Picard–Lefschetz Theory,** Mathematical Surveys and Monographs, Vol. 97, American Mathematical Society, 2002, 324+xi pp.

Articles in Refereed Journals and Periodicals

[18] **Asymptotics of exponential integrals, Newton diagram and classification of the minima points,** Funct. Anal. and its Appl., 11:3 (1977), p. 1–11.

[19] **On the affinneness of normal forms of the $\mu =$ const strata of germs of smooth functions,** Funct. Anal. and its Appl., 12:3 (1978), p. 72–73.

[20] **Asymptotics of exponential integrals in the complex domain,** Funct. Anal. and its Appl., 13:4 (1979), p. 1–12.

[21] **Characteristic classes of Lagrange and Legendre manifolds dual to singularities of caustics and wave fronts,** Funct. Anal. and its Appl., 15:3 (1981), p. 10–22.

[22] **Self-intersections of wave fronts and Legendre (Lagrange) characteristic numbers,** Funct. Anal. and its Appl., 16:2 (1982), p. 68–69.

[23] **Translator's note to the article of M.F. Atiyah, R. Bott and L. Gårding "Lacunas for hyperbolic differential operators with constant coefficients. II",** Uspekhi Matem. Nauk, 39:3 (1984), p. 221–223. (In Russian.)

[24] **Shaprness and local Petrovsky condition for hyperbolic operators with constant coefficients,** Izvestija Ac. Sci. USSR, Ser. Math., 50:2 (1986), p. 242–283.

[25] **Behavior of general hypergeometric functions in the complex domain** (with I.M. Gel'fand and A.V. Zelevinsky), Russian Math. Doklady, 290:2 (1986), p. 277–281.

[26] **General hypergeometric functions on complex grassmannians** (with I.M. Gel'fand and A.V. Zelevinsky), Funct. Anal. and its Appl., 21:1 (1987), p. 23–38. Revised version in: I.M. Gelfand, Selected works, vol. 3 (Springer-Verlag).

[27] **Stable cohomology of complements of discriminant manifolds of singularities of holomorphic functions,** Uspekhi Matem. Nauk, 42:2 (1987), p. 219–220. (Russian, English translation in Russian Math. Surveys 42:2 (1987).

[28] **Cohomology of braid groups and complexity of algorithms,** Funct. Anal. and its Appl., 22:3 (1988), p. 15–24.

[29] **On the topology of spaces of functions without complicated singularities,** Uspekhi Matem. Nauk, 44:3 (1989), p. 149–150. (In Russian, English transl. in Russian Math. Surveys 44:3 (1989).)

[30] **Newton's Principia read 300 Years later** (with V.I. Arnold), Notices Amer. Math. Soc., 36:9 (1989), p. 1148-1154.

[31] **Topological complexity of algorithms of approximate solution of systems of polynomial equations**, Algebra i Analiz, 1:6 (1989), p. 98-113; English transl. in Leningrad Math. J., 1 (1990), p. 1401–1417.

[32] **Topology of spaces of functions without complicated singularities**, Funct. Anal. and its Appl., 23:4 (1989), p. 24–36.

[33] **On the numbers of real and complex moduli of singularities of smooth functions and realizations of matroids** (with V.V. Serganova), Matem. Zametki, 49:1 (1991), p. 19–27. (Russian, Engl. transl. in Math. Notes, 49:1 (1991), p. 15–20.)

[34] **On the spaces of functions interpolating at any k points**, Funct. Anal. and its Appl., 26:3 (1992), p. 72–74.

[35] **Geometry of the local lacunas of hyperbolic operators with constant coefficients,** Mat. Sbornik, 183:1 (1992), 114-129. (Russian, English transl. in Russian Acad. Sci. Sbornik Math. 75 (1993), p. 111–123.)

[36] **A geometric realization of the homology of classical Lie groups, and complexes, S-dual to the flag manifolds,** Algebra i Analiz 3:4 (1991), p. 113–120. (Russian, English transl. in St.-Petersburg Math. J., 3:4 (1992), p. 809–815.)

[37] **Stratified Picard–Lefschetz theory,** Selecta Math., New Ser., 1:3 (1995), p. 597–621.

[38] **On spaces of polynomial knots**, Matem. Sbornik, 187:2 (1996), p. 37–58.

[39] **Topological complexity and reality**, Matem. Zametki (Math. Notes), 1996, 60:5, p. 270–280.

[40] **Holonomic links and Smale principles for multisingularities,** J. of Knot Theory and its Ramifications, 6:1 (1997), p. 115–123.

[41] **Decision trees for orthants,** Information Processing Letters, 62 (1997), p. 265–268.

[42] **Homology of spaces of homogeneous polynomials without multiple roots in $\mathbf{R}^2$**, Proc. Steklov Math. Inst., vol. 221, 1998, p. 143–148.

[43] **On k-neighborly submanifolds in $\mathbf{R}^N$**, Topological Methods in Nonlinear Analysis, 11:2 (1998), p. 273–281.

[44] **How to calculate homology groups of spaces of nonsingular algebraic projective hypersurfaces**. Proc. Steklov Math. Inst., vol. 225, 1999, p. 121–140.

[45] **On a problem by M. Kazarian,** Funct. Anal. and its Appl. 33:3 (1999), p. 73–75.

[46] **Topological order complexes and resolutions of discriminant sets,** Publications de l'Institut Mathématique Belgrade, Nouvelle série, t. 66(80), 1999, 165–185.

[47] **Combinatorial formulas for cohomology of spaces of knots,** Moscow Math. J., 2001, **1**:1, 91–123.

[48] **Homology of spaces of knots in any dimensions,** Philos. Trans. London Royal Society A, **359**:1784 (2001), 1343–1364. (Proceedings of the Royal Society Discussion Meeting "Topological Methods in the Physical Sciences", November 15–16, 2000).

[49] **Topology of plane arrangements and their complements,** Russian Math. Surveys, **56**:2, 2001, 167–203.

[50] **Spaces of Hermitian matrices with simple spectra and their finite-order cohomology.** Moscow Math. Journal, 3:3 (2003), 1145–1165.

[51] **Combinatorial computation of combinatorial formulas for knot invariants**, Transact. of Moscow Math. Society **66** (2005), 3–94, to appear.

[52] **First degree invariants for spatial self-intersecting curves**, Izvestiya Russian Ac. Sci., 2005, to appear.

Articles in Refereed Collections of Works

[53] **Estimates for complex exponential integrals,** Some Questions of Mathematics and Mechanics; Moscow State Univ. Press, 1981, p. 76–77. (In Russian.)

[54] **Sharp and diffuse fronts of hyperbolic equations,** Itogi Nauki i Tekhn. VINITI. Fundamental Directions. Moscow, VINITI, vol. 31, 1988, p. 246–257. (Russian, English transl. in: Encycl. Math. Sci., vol. 31 (Partial differential equations II) Springer–Verlag, Berlin & New York.)

[55] **Stable cohomology of complements to the discriminants of deformations of singularities of smooth functions,** Current Problems of Math., Newest Results, vol. 33, Itogi Nauki i Tekhniki, VINITI, Moscow, 1988, p. 3–29; (Russian, English transl. in J. Soviet Math. 52:4 (1990), p. 3217–3230.)

[56] **Characteristic classes of singularities,** Theory of Operators in Functional Spaces, Kuybyshev, 1989, p. 15–29; English transl. in Amer. Math. Soc. Transl. (2) vol. 153, 1992, p. 11–23.

[57] **Lacunas of hyperbolic partial differential operators and singularity theory,** Theory of Operators in Functional Spaces, Kuybyshev, 1989, p. 30–43; English transl. in Amer. Math. Soc. Transl. (2) vol. 153, 1992, p. 25–37.

[58] **Topology of complements to discriminants and loop spaces,** Theory of Singularities and its Applications (V.I. Arnold, ed.), Advances in Soviet Math., vol. 1 (1990), p. 9–21 (AMS, Providence, RI).

[59] **Cohomology of knot spaces,** Theory of Singularities and its Applications (V.I. Arnold, ed.), Advances in Soviet Math. Vol. 1 (1990), p. 23–69 (AMS, Providence, RI).

[60] **Knot invariants and singularity theory**, In: Singularity Theory (Trieste, 1991), Le D.T., K. Saito and B. Teissier, eds., World Sci. Publishing, River Edge, NJ, 1995, p. 904–919. (Proceedings of the Colloquium in Singularity Theory, Trieste, ICTP, Aug. 19–Sep. 06, 1991).

[61] **Cohomology of braid groups and complexity,** From Topology to Computation: Proc. of the Smalefest conference, M. Hirsch, J. Marsden and M. Shub, eds.; Springer-Verlag, Berlin and New York, 1993, p. 359–367.

[62] **The Smale–Hirsch principle in the catastrophe theory,** From Topology to Computation: Proc. of the Smalefest conference, M. Hirsch, J. Marsden and M. Shub, eds.; Springer–Verlag, Berlin and New York, 1993, p. 117–128.

[63] **Complexes of connected graphs,** The I.M. Gel'fand's mathematical seminars 1990–1992, L. Corvin, I. Gel'fand, J. Lepovsky, Eds.; Birkhäuser, Basel, 1993, p. 223–235.

[64] **Invariants of knots and complements of discriminants,** in: Developments in Math., the Moscow school (V. Arnold, M. Monastyrski, eds.), Chapman & Hall, 1993, p. 194–250.

[65] **Invariants of ornaments,** Singularities and Bifurcations (V.I. Arnold, ed.), Advances in Soviet Math., vol. 21 (1994), p. 225–262 (AMS, Providence, R.I.).

[66] **Ramification in integral geometry and monodromy of complex links**, J. of Mathematical Sciences, 83:4 (1997), p. 554–558.

[67] **Topology of discriminants and their complements,** Proceedings of the Intern. Congress of Math. (Zürich, 1994); Birkhäuser Verlag, Basel; 1995, p. 209–226.

[68] **Topological complexity of root-finding algorithms** Proc. of the AMS/SIAM 1995 Summer Seminar; AMS, 1996, p. 831–856.

[69] **Stratified Picard-Lefschetz theory with twisted coefficients,** in: Topics in Singularity Theory, V.I. Arnold's 60-th anniversary volume, AMS Translations, ser. 2, vol.180; Advances in Math. Sci. 34, 1997, p. 241–255.

[70] **Monodromy of complete intersections and surface potentials** in: Singularities. The Brieskorn Anniversary Volume. Progress in Mathematics, Vol. 162. Birkhäuser Verlag, Basel-Boston-Berlin, 1998, p. 205–237.

[71] **On invariants and homology of spaces of knots in arbitrary manifolds,** in: Topics in Quantum Groups and Finite-Type Invariants. Mathematics at the Independent University of Moscow. B. Feigin and V. Vassiliev, eds. AMS Translations. Ser. 2. Vol. 185. Advances in the Mathematical sciences. AMS, Providence RI, 1998, p. 155–182.

[72] **Topology of two-connected graphs and homology of spaces of knots,** In: "Differential and symplectic topology of knots and curves," AMS Transl. Ser. 2. Vol. 190. (S.L. Tabachnikov, ed.). AMS, Providence RI, 1999, p. 253–286.

[73] **Homology of i-connected graphs and invariants of knots, plane arrangements, etc.** Proc. of the Arnoldfest Conference, Fields Inst. Communications, Vol. **24**, p. 451–469, AMS, Providence RI, 1999.

[74] **On finite-order invariants of triple points free plane curves,** In: AMS Transl. Ser. 2. Vol. 194. Volume dedicated to the 60-th birthday of D.B. Fuchs, (A. Astashkevich and S. Tabachnikov, eds.) AMS, Providence RI, 1999, p. 275–300.

[75] **Mathematical models of catastrophes. Control of catastrophic processes** (with V.I. Arnold, A.A. Davydov, and V.M. Zakalyukin), in: UNESCO Encyclopaedia of Life Support Systems, EOLSS Publishers Co. Ltd., ??.

[76] **Resolutions of discriminants and topology of their complements**, in: New Developments in Singularity Theory, D. Siersma, C.T.C. Wall and V.M. Zakalyukin, eds., Kluwer Academic Publ., Dorderecht, 2001, 87–115.

[77] **Ramified Integrals and Picard–Lefschetz Theories,** Globus. General Mathematical Seminar of Independent University of Moscow. Vol. 1. (M.Tsfasman and V.Prasolov, eds.). MCCME, 2004, 22–58.

[78] **Algorithms for the combinatorial realization of cohomology classes of spaces of knots**, in: Fundamental Mathematics Today. In honor of the 10th anniversary of the Independent University of Moscow (S.Lando and O.Sheinman, eds.). Moscow, MCCME, 2003, pp. 10–31.

[79] **Combinatorial formulas for cohomology of spaces of knots**, in: Advances in Topological Field Theory (J.Bryden, ed.) Springer-Verlag, 2004.

Selected Preprints and Manuscripts

[80] **Singularities of caustics and their applications to the investigation of asymptotics of exponential integrals and Lagrange manifolds**. Ph.D. Thesis, Moscow State Univ., 1981, 148 p. (In Russian.)

[81] **Singularities of caustics and their applications to the investigation of asymptotics of exponential integrals and Lagrange manifolds**. Summary of [80], 13 p.

[82] **Homological invariants of knots: algorithms and calculations,** Preprint No.90, Inst. of Applied Math., 23 p. (In Russian)

[83] **Complements of discriminants of smooth maps,** Dr. of Sciences Thesis, Moscow, 1990, 300 p. (In Russian).

[84] **Complements of discriminants of smooth maps,** Summary of [83], Moscow, 1990, 29 p. (In Russian).

[85] **Complements to discriminants of smooth mappings**, ICTP, Trieste, Preprint SMR.567/21, 1991, 17 p.

[86] **Knot invariants and singularity theory**, ICTP, Trieste, Preprint SMR.567/10, 1991, 16 p.; see also [60].

[87] **Stable homotopy type of the complement to affinne plane arrangement,** preprint, 1991, 4 p.

[88] **Invariants of ornaments,** Maryland Univ., March 1993; 30 p.

Abstracts and Summaries of Talks

[89] **Lagrange singularities and Lagrange characteristic classes,** Uspekhi Matem. Nauk, 37:4 (1982), p. 96–97. (In Russian.)

[90] **Local Petrovsky condition and Picard-Lefschetz theory,** Uspekhi Matem. Nauk, 39:2 (1984), p. 219–220. (In Russian.)

[91] **Sharp fronts of hyperbolic operators with constant coefficients,** Uspekhi Matem. Nauk, 41:4 (1986), p. 162. (In Russian.)

[92] **Topology of complements of discriminant varieties and complexity of algorithms**, Uspekhi Matem. Nauk, 1987, 42:5, p. 203.

[93] **Newton's nonsquarability theorem for multidimensional bodies,** Proc. 13 School on the Operator Theory. Kuybyshev, 1988, p. 42–43. (In Russian).

[94] **Newton's Principia 300 years later,** Uspekhi Matem. Nauk, 1989, **44**(6), p. 167 (with V.I. Arnold).

[95] **Topology of spaces of functions without complicated singularities**, Modern algebraic and funct.-analytic methods in analysis. Voronezh univ., Voronezh, 1990, p. 52–53 (in Russian).

[96] **Topology of discriminants and their complements,** in: Book of Abstracts of the Intern. Congress of Math., Zürich, 1994, p. 12–14.

[97] **Algebraicity of surface potentials and monodromy of complete intersections,** Math. Inst. Oberwolfach, Tagungsbericht 27/1996.

[98] **Invariants of knots in arbitrary manifolds,** Uspekhi Matem. Nauk, 1998, 53:2, p. 171.

[99] **Topological order complexes and topology of discriminants,** Book of abstracts of the international conference "Geometric Combinatorics" (Kotor, Yugoslavia, Aug.28–Sept.03, 1998), p. 11, 1998.

[100] **Topological order complexes and topology of discriminants,** Book of abstracts of the international conference dedicated to 90-th anniversary of L.S. Pontryagin (Moscow, Aug.31–Sept.06, 1998), Vol. Algebra, Geometry and Topology, Moscow, MSU, p. 106–108.

[101] **Order complexes of singular sets and topology of spaces of nonsingular projective varieties,** Abstracts of the international conference "Topology and Dynamics. Rokhlin memorial" (St. Petersburg, Russia, Aug. 19–25, 1999), p. 72–73.

[102] **Order complexes of singular sets and topology of spaces of nonsingular projective varieties**, Math. Inst. Oberwolfach, Tagungsbericht 19/1999.

[103] **Collections of planes and their complements**, Uspekhi Matem. Nauk (Russian Math. Surveys), 2001, No.?, ?? .

[104] **Theory of lacunas and Petrovskii condition for hyperbolic operators**, Book of Abstracts of the International Conference "Differential Equations and Related Topics" dedicated to the Centenary Anniversary of I.G. Petrovskii, Moscow University Press, 2001, p. 18.

[105] **Combinatorial formulas for cohomology of spaces of knots,** Abstracts of the International Conference "Fundamental Mathematics Today" dedicated to the 10-th anniversary of the Independent University of Moscow, Moscow, December 2001, p. 40.

[106] **Geometry of Discriminants and Classical Homotopy Topology,** in: "Kolmogorov and Contemporary Mathematics. Abstracts. Russian Ac. of Sci., Moscow State Univ.; p. 785.

[107] **Cohomology of knot spaces and their combinatorial formulas**, Uspekhi Matem. Nauk (Russian Math. Surveys), 2004, 59:2, p. 202.

[108] **On the cohomology of spaces of self-intersecting curves**, Abstracts of International Conference "Combinatorial Methods in Physics and Knot Theory", Moscow, Independent University of Moscow, p. 16.

Other

[109] **The mathematical legacy of "Lacunas for hyperbolic equations"**, in: Raoul Bott, Selected Papers. Vol. 2: Differential Operators. Robert D. MacPherson, Ed.; Birkhäuser; Boston, Basel, Berlin; 1994, p. xxiii - xxviii.

[110] **Introduction** to the book: Topics in Singularity Theory. V.I. Arnold's 60-th Anniversary Collection, AMS Translations, Ser. 2, vol. 180. AMS, Providence RI, 1997, p. ix–xiii (with A.G. Khovanskii and A.N. Varchenko).

[111] **V.I. Arnold: to the 60-th anniversary**. Uspekhi Matem. Nauk, 1997, 52:5, p. 235-255 (With D.V. Anosov, A.A. Bolibruch, A.A. Gonchar, M.L. Gromov, S.M. Gusein-Zade, L.D. Faddeev, Yu.S. Ilyashenko, B.A. Khesin, A.G. Khovanskii, M.L. Kontsevich, V.V. Kozlov, Yu.I. Manin, A.I. Neishtadt, S.P. Novikov, Yu.S. Osipov, M.B. Sevryuk, Ya.G. Sinai, A.N. Tyurin, A.N. Varchenko, A.M. Vershik, and V.M. Zakalyukin).

[112] **Introduction** to the book: Geometry of Differential Equations, AMS Translations, Ser. 2, vol. 186. AMS, Providence RI, 1998, p. ix–xi (with A.G. Khovanskii and A.N. Varchenko).

[113] **Comments to some 27 problems**, in: V.I. Arnold's Problems, Moscow, Phasis 2000, pp. 184, 185, 207, 226, 227, 236, 238, 277, 278, 279, 284, 288, 292-293, 300, 310, 312-314, 315-316, 327, 330-331, 337-338, 339, 351, 366-367, 418 (in Russian).

[114] **Comments to some 32 problems,** in extended English version of [113], Phasis and Springer, 2004.

[115] **Theory of Petrovskii lacunas of hyperbolic equations**, a chapter in the V.I.Arnold's book "Partial Differential Equations", Phasis, to appear, 9 pp.

[116] **Role of Mathematics, as They Think of It**, Notices AMS, September 2004, 870–871

I have also authored and co-authored six articles on applied mathematics.

Books edited and co-edited

[1] **Proc. of the 13 All-Union School on the Operator Theory,** Kuybyshev, 1988. (In Russian).

[2] **Russian translation of the Proceedings of the Bourbaki Seminare (1989)**, Moscow, Mir Publishers, 1995.

[3] **Russian translation of the Proceedings of the Bourbaki Seminar (1990),** Moscow, Mir Publishers, 1996.

[4] **Russian translation of the Proceedings of the Bourbaki Seminar (1991),** Moscow, Mir Publishers, 1998.

[5] **Topics in Singularity Theory. V.I. Arnold's 60-th Anniversary Collection,** AMS Translations, Ser. 2, vol. 180. AMS, Providence RI, 1997.

[6] **Proceedings of the International Geometrical Colloquium (Moscow, Mai 1993). Parts I-III.** VINITI, ser. Contemporary Mathematics and its Applications. Thematic Surveys, vol. 20, 21, 22. Moscow, VINITI, 1995–1996. (In Russian, Engl. transl. in J. of Mathematical Sciences, Plenum Press, vol. 83:2–83:4 ??, 1996–1997.)

[7] **Topics in Quantum Groups and Finite-Type Invariants. Mathematics at the Independent University of Moscow,** AMS Translations. Ser. 2. Vol. 185. Advances in the Mathematical sciences. AMS, Providence RI, 1998.

[8] **Geometry of Differential Equations,** AMS Translations, Ser. 2, vol. 186. AMS, Providence RI, 1998, 194 p.

[9] **Russian transl. of "Mathematics: Frontiers and Perspectives"**, M.: Phasis, ???.

[10] **Russian translation of "Introduction to Combinatorial Torsions"** by V.Turaev, M., MCCME, 2004

[11] **Selected Works of N.H. Abel,** MCCME, Moscow, 2005 (to appear).

[12] **Low-Dimensional Topology series** by Springer-Verlag (with R.V. Gamkrelidze). Vols. 140 (2003), 141 (2003), 142 (2004)

RESEARCH SUMMARY

1. Topology of spaces of knots and plane curves.

In [59], [9] I constructed spectral sequences which calculate, in principle, all cohomology groups of spaces of knots and links in $\mathbf{R}^n$, $n \geq 3$, in particular (for $n = 3$) the knot invariants. The invariants derived from the efficient part of these calculations (so-called finite-order invariants or Vassiliev invariants) dominate all known polynomial invariants of links and Milnor's higher linking numbers. For $n > 3$ this spectral sequence gives immediately a calculation of all cohomology groups of the space of smooth links in $\mathbf{R}^n$. Later, using the concept of a *conical resolution* (cf. [36]) I improved the general spectral sequence and calculated the homology type of spaces of knots realized by polynomial embeddings $\mathbf{R}^1 \to \mathbf{R}^n$ of degree ≤ 4, see [38]. In my 1991 lecture notes [60], see also [63], I noticed a similar spectral sequence calculating homotopy invariants of links.

In [71] I extended the main spectral sequence to one calculating the finite-order invariants of knots (and cohomology classes of higher dimensions of spaces of knots) in arbitrary 3-manifolds, including non-orientable and reducible ones, and presented examples of singular knot invariants, which cannot be integrated to usual knot invariants. (For orientable reducible manifolds, these invariants coincide with the ones studied previously by E. Kalfagianni.) The knot invariants arise as the elements of final terms $E_\infty^{-i,i}$ of this spectral sequence, however the spectral sequence itself (especially its higher differentials) is an invariant of the ambient 3-manifold. Also I showed that there are *integer-valued* finite-order knot invariants in all sufficiently complicated *non-orientable* 3-manifolds, distinguishing infinitely many different knots in them.

In [63], [65], [10] I started the study of finite-order invariants of *ornaments*, i.e. collections of closed plane curves without triple intersections. Using a similar spectral sequence I found a large class of elementary invariants of such objects; the explicit calculation of this sequence leads to many natural problems in the modern homological combinatorics, cf. [72].

This theory applies perfectly to the classification of one-component plane immersed curves without triple intersections, developed by Arnold and extended by Polyak, Viro, Tabachnikov, Shumakovich, and Merkov, and also to more complicated theory of invariants of triple-points free plane curves with allowed singular points. This theory leads naturally to the study of *triangular diagrams* and *connected and two-connected hypergraphs* in the same way as the parallel knot theory leads to the calculus of chord diagrams and connected graphs. E.g., the simplest invariant of triple points free singular curves is of order 4 and is depicted by the triangular diagram  in the same way as the simplest knot invariant (of order 2) corresponds to the crossed 2-chord diagram $\bigoplus$, see [74].

In [72] I calculated all groups of order ≤ 3 cohomology classes of spaces of knots in $\mathbf{R}^n$, $n \geq 3$. (In the case of odd n one of these classes, having order 3 and dimension $3n - 8$ was found by my students D. Teiblum and V. Turchin in a computer experiment.) The very first of them is a $\mathbf{Z}_2$-valued $(n-2)$-dimensional cohomology class of order 1 in the space of compact knots in $\mathbf{R}^n$, defined as the linking number with the set of singular maps $S^1 \to \mathbf{R}^n$, gluing together some two *opposite* points of S^1. For even n similar class generates also the *integral* $(n-2)$-dimensional cohomology group (which is free cyclic). For $n = 3$ this class is nontrivial already in the component of unknots and proves that this component is not simply-connected.

In [47], [51] I have introduced a homological calculus for finding explicit combinatorial formulas for cohomology classes of spaces of knots in $\mathbf{R}^n$, generalizing the Polyak–Viro formulas for knot invariants in $\mathbf{R}^3$. In particular, in [47] I gave such an expression for the Turchin-Teiblum cocycle (reduced mod 2) and for all integral cohomology classes of orders ≤ 2. These expressions gave the first proof of the non-triviality of the Turchin-Teiblum cocycle in the space of long knots in $\mathbf{R}^3$, and also of an 3-dimensional cohomology class of degree 2 in the space of compact knots in $\mathbf{R}^3$. In [51] I described a purely combinatorial algorithm of computing all finite-type knot invariants. This algorithm deals not with the planar pictures like the knot diagrams but with easily encodable objects like the chord diagrams; therefore it is ready for the efficient computer realization.

In [52], I have calculated all first degree cohomology classes of spaces of smooth generic immersions $\mathbf{R}^1 \to \mathbf{R}^n$, $n \geq 3$, with a standard behavior at infinity and prescribed transverse self-intersections; in particular all first degree invariants of such immersions for $n = 3$. Also, I have proved that any such cohomology class can be represented by a combinatorial formula generalizing the Gauß formula for linking numbers, containing only half-integer coefficients, and constructed the unique (and non-trivial) topological obstruction to the existence of similar combinatorial formulas with integer coefficients. As a corollary, I have proved that Polyak–Viro combinatorial formulas of certain knot invariants of degree 4 have necessarily non-integer coefficients. These homological calculations have led me to the formulation of an elementary criterion of planarity of Euler graphs, which was proved later by V.O. Manturov.

In [40] I have proved that any knot or link in $\mathbf{R}^3$ is isotopy equivalent to a holonomic one (i.e. given by the 2-jet extension of a differentiable function) and that the space of holonomic links in $\mathbf{R}^n$, $n > 3$, is homotopy equivalent to the space of all links. Later, J. Birman and N. Wrinkle have proved that any two holonomic knots, which are isotopy equivalent as usual knots, can be connected by a path in the space of holonomic knots.

2. Geometric combinatorics, theory of plane arrangements and matroids.

In 1991, simultaneously with G. Ziegler and R. Živaljević, I proved that the stable homotopy type of the complement of an arbitrary arrangement of affine planes depends only on the dimensions of (multiple) intersections of these planes; moreover, I gave an explicit expression for the homotopy type of the (Spanier–Whitehead dual to this complement) one-point compactification of the arrangement in the terms of these dimensions, see [87], [63]. The Goresky-MacPherson formula for the cohomology of the complement of the arrangement follows immediately from this expression.

In the joint work with V. Serganova, [33], we constructed a matroid of rank three such that the space of its real realizations is non-empty but the dimension of this space is strictly less than that of the similar space of complex realizations. This example allowed us to prove some old problems in Singularity Theory, see § 6 below.

In the joint work with I.M. Gel'fand and A.V. Zelevinskii [26] we calculated the homology groups of local systems on complements of a large class of complex hyperplane arrangements and gave an explicit geometric realization of their generators. (See also [13])

In [59], [82], [63] I started the study of the *complex of connected graphs* and calculated its homology groups, which are an important ingredient in the calculation of homology groups of spaces of knots. In [63], [65], [74] I started the study of complexes of *connected hypergraphs*, playing a similar role in the theory of generic plane curves, and in [72], [12] the study of complexes of *two-connected graphs,* providing an alternative approach to the homological study of knot spaces. These problems initiated a large series of works by specialists in Geometric Combinatorics (A. Björner, V. Welker, R. Simion, J. Shareshian, V. Turchin, a.o.).

In [36], [46] I have introduced the *topological order complexes* of topologized partially ordered sets. This concept proved to be a very powerful tool in calculating homology groups of various discriminant spaces.

In [42] I used this notion to calculate the homology of some discriminant subvarieties in the spaces of polynomial functions $\mathbf{R}^2 \to \mathbf{R}^1$. In [44], also using this method, I gave an algorithm of computing homology groups of spaces of nonsingular algebraic hypersurfaces in $\mathbf{CP}^n$ and calculated such groups for $d + n \leq 6$; e.g. the Poincaré polynomial of such a space with $d = 4, n = 2$ is equal to $(1 + t^3)(1 + t^5)(1 + t^6)$. In [50] I applied this method to the study of spaces of Hermitian operators with simple spectra. For an overview of this method and these results, see also [100], [46], [76].

In [36] I proved that the naturally topologized order complex of all proper subspaces in a vector space over $\mathbf{R}$, $\mathbf{C}$ or $\mathbf{H}$ is homeomorphic to a sphere.

3. Complexity theory. The *topological complexity* of a computational problem is the minimal number of branchings (operators IF) in algorithms solving it. Generalizing a work of S. Smale, I found the best known and asymptotically sharp two-side estimate of the topological complexity of approximate solution of polynomial equations in one complex variable, see [28], [9].

In [39] I proved that the topological complexity of approximate solution of real polynomial equations of degree d is equal to $d/2$ if d is even, and for odd $d \geq 3$ it also is positive (and is equal to 1 if $d = 3$ or 5).

In [31], [9] I found best known estimates for complexity of approximate solution of similar problems on solving systems of polynomial equations in $\mathbf{C}^n$; it turned out that in all important cases the complexity is asymptotically proportional to the dimension of the space of systems.

In [41] I prove the following extension of the M. Rabin's theorem about recognizing the main orthant in $\mathbf{R}^n$: the depths of analytic decision trees recognizing the union of any r orthants in $\mathbf{R}^n$ are estimated from below by the number $n - \text{ord}_2 r$ (where $\text{ord}_2 r$ is the number of twos in the primary decomposition of r), in particular by n if r is odd. As a corollary I prove that for any $d = 2^q$ and $l \in [1, d/2]$ the minimal depth of any analytic decision tree recognizing the set of real polynomials of degree d having at least $2l - 1$ real roots is no less than $d/2$.

A submanifold $M \subset \mathbf{R}^n$ is *r-neighborly,* if for any r points of M there is a hyperplane, supporting M and touching it at exactly these r points. In [43] I prove that the minimal dimension of the space $\mathbf{R}^n$, containing a stably r-supported k-dimensional submanifold, is asymptotically no less than $2kr - k$. This is the first general lower estimate in this problem, posed by M. Perles in the 1970-ies.

In [12], [34] I found an asymptotically sharp lower estimate of the minimal dimension of functional spaces whose elements interpolate any function $\mathbf{R}^n \to \mathbf{R}$ at any k points: if n is a power of 2 then this number lies in the interval $[k + (n - 1)(k - d(k)), k(n + 1)]$, where $d(k)$ is the number of ones in the binary representation of k. Similar estimates for functions on general manifolds are expressed in the terms of characteristic classes of configuration spaces.

4. Symplectic topology. In the works [21], [22], [1] I have constructed the universal chain complexes of singularities and multisingularities of wave fronts and caustics, which provide characteristic classes of Lagrangian and Legendrian manifolds, generalizing the Maslov index. They allow us to prove numerous restrictions on the coexistence and numbers of singular points on such manifolds.

Recently M. Kazarian obtained a strong generalization and spectacular applications of these techniques to different problems of differential and symplectic topology.

In [45] I have proved that any compact group of symmetries of any real function singularity with finite Milnor number and zero 2-jet is discrete. This proves (modulo the results of Kazarian) that all rational Lagrange characteristic classes can be expressed in the terms of cohomology groups of our universal complex.

5. Theory of lacunas of hyperbolic PDE's. In 1983 I proved that the sharpness (= the regularity of fundamental solutions) at points of wave fronts of almost all hyperbolic operators is equivalent to the local topological Petrovskiĭ condition (it was conjectured by Atiyah, Bott and Gårding, who introduced this condition in 1973) and demonstrated that for very degenerate fronts similar conjecture is false, see [90], [23], [24], [11]. I also reduced the verification of the Petrovskiĭ condition to the computation of standard characteristics of singularities of functions, and found all local lacunas (domains of sharpness) close to "simple" (i.e. of types A_k, D_k, E_k) singularities of wave fronts, in particular close to all points of generic fronts in $\mathbf{R}^n$, $n \leq 7$, see [24], [7], [11]. In [35] I gave a simple geometrical characterization of local lacunas close to such singularities of fronts.

See also the last paragraph of the next § 6.

6. Theory of singularities. In my works on singularity theory several problems of the V.I. Arnold's 1976 and 1979 published lists of problems are solved, namely

- In our joint work with V. Serganova [33] we proved that the number of real moduli of a singularity of a real function can be strictly less than the number of complex moduli of its complexification; this provides also a new example of a non-smooth $\mu =$ const stratum of an isolated singularity.
- In [55] I proved the stable irreducibility of classes of singularities and multisingularities in the parameter spaces of versal deformations of singularities of complex functions.
- In [55], [58] I calculated stable cohomology rings of complements of discriminants and caustics of isolated singularities in $\mathbf{C}^n$. These rings are naturally isomorphic to the cohomology rings of the iterated loop spaces $\Omega^{2n}S^{2n+1}$ and $\Omega^{2n}\Sigma^{2n}U(n)/O(n)$ respectively. See also [27], [9], [10].

Also I wrote a FORTRAN algorithm enumerating topologically distinct morsifications of singularities of real functions. Using this algorithm, I found new local lacunas close to many singularities of wave fronts and proved the absence of such lacunas for some other singularities. See [11], [7].

7. Dynamical systems. In 1979 I constructed the first example of a multidimensional pursuit problem, the dimension of whose attractor is twice greater than that of the target manifold of the problem; see pp. 219-220 of the expository article of Yu.S. Il'yashenko in Selecta Math. Sovetica, 1992, 11:3.

8. Generalized hypergeometric functions. In the joint works with I.M. Gel'fand and A.V. Zelevinskiĭ [25], [26] we found the numbers of solutions of generalized hypergeometric systems on all sufficiently general strata of Grassmann manifolds, and proved that all of these solutions have integral representations.

9. Integral geometry and Picard–Lefschetz theory. Newton proved that the area cut by a line from the domain bounded by a smooth plane curve cannot be an algebraic function of the cutting line. In 1987 I proved a similar result for all convex domains in even-dimensional spaces, and found many obstructions to algebraicity in the odd-dimensional case (so that the Archimedes' example of an ellipsoid in $\mathbf{R}^{2n+1}$ becomes an exceptional phenomenon); these obstructions are formulated in terms of the local geometry of the complexification of the boundary of the domain. The proofs are based on methods of the (generalized) Picard–Lefschetz theory, see [30], [7], [11].

Studying the ramification of (the analytical continuations of) the volume function I found new generalized Picard–Lefschetz formulae, reducing the ramification of (both standard and intersection) homology groups of stratified singular varieties to the similar problem concerning the transversal slices of their strata, see [7], [37], [11], as well as similar reduction formulas for the ramification of homology (generally speaking, with coefficients in non-constant local systems) of complements of algebraic varieties depending on parameters, see [69].

10. Differential topology. In 1988 I proved the "homological Smale–Hirsch principle" for the spaces of smooth functions $M \to \mathbf{R}^n$ without complicated singularities (i.e. without singularities defining subsets of codimension ≥ 2 in functional spaces); this theorem reduces the computation of the homology groups of such spaces to those of the corresponding spaces of admissible sections of the jet bundle $J^{\cdot}(M, \mathbf{R}^n) \to M$.

In 1985, working on a related problem (mentioned in the third paragraph of § 6) I constructed a spectral sequence calculating the homology groups of such spaces, see [27], [55]; two simplest particular cases of this spectral sequence (corresponding to singularity classes defined by 0-jets of maps) coincide with the Adams spectral sequence for loop spaces and the Anderson spectral sequence for spaces of maps of low-dimensional spaces into highly-connected ones.

Using similar techniques, I proved in 1991 that the space of systems of k monic polynomials of degree d in $\mathbf{C}^1$ (or $\mathbf{R}^1$) without common roots is stably homotopy equivalent to the space of monic polynomials of degree $d \cdot k$ in $\mathbf{C}^1$ (respectively, $\mathbf{R}^1$) without roots of multiplicity k; this extends a theorem of F. Cohen, R. Cohen, B. Mann and J. Milgram establishing similar equivalence in the case $k = 2$. See [85], [64], [10].

In [42] for any d, k I calculated the cohomology group of the space of homogeneous polynomials $\mathbf{R^2} \to \mathbf{R^1}$ of degree d having no roots of multiplicity $\geq k$ in $\mathbf{RP}^1$. For $k = 2$ this problem gives us the first known example of the situation when finite-order invariants of something (in this case of smooth functions $S^1 \to \mathbf{R}^1$ without double zeros) do not constitute a complete system of invariants.

See also § 1 and [59].

11. Potential theory. Extending the famous theorems of Newton and Ivory about the potentials of spherical and elliptic layers, Arnold proved in 1982 that a hyperbolic layer in $\mathbf{R}^n$ does not attract a particle in the hyperbolicity domain. In [11], [70] I investigated for which d and n the potential of such a layer coincides with algebraic functions in other domains. I proved that for $n = 2$ or $d = 2$ it is always so, and for $(n \geq 3)\&(d \geq 3)\&(n+d \geq 8)$ the potential of a generic hyperbolic surface of degree d in $\mathbf{R}^n$ is not algebraic in any domain other than the hyperbolicity domain.

For all values of n and d I reduced this problem to the calculation of a certain subgroup of the monodromy group of some singularity of a complete intersection of codimension 2 in $\mathbf{C}^n$. Using this reduction, W. Ebeling has proved the similar non-algebraicity statement for remaining three cases $(d, n) = (3, 3), (3, 4)$ and $(4, 3)$.

12. Topology of Lie groups. In [36] I constructed natural conical resolutions of determinant subvarieties in the spaces $GL(\mathbf{K}^n)$, $\mathbf{K} = \mathbf{R}, \mathbf{C}$ or $\mathbf{H}$. This construction (and the related natural filtration) gives an immediate realization of Miller's splitting of the homology of classical Lie groups into the homology of Grassmann manifolds, and allows to prove that the naturally topologized order complex of the set of all subspaces in $\mathbf{K}^n$ is homeomorphic to a sphere.