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ABSTRACTS

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Justification and evidence-based knowledge

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Plato's much celebrated tripartite definition of knowledge as *justified true belief* (JTB) is generally regarded as a set of necessary conditions for the possession of knowledge. Due to Hintikka, the "true belief" components have been fairly formalized by means of modal logic and its possible worlds semantics. Despite the fact that the justification condition has received the greatest attention in epistemology, it lacked a formal representation.

We introduce justification into formal epistemology by combining Hintikka-style epistemic modal logic with justification calculi arising from the logic of proofs. A formal epistemic logic with justification contains assertions of the form $\Box F$ (*F is known*), along with those of the form $t{:}F$ (*t is a justification for F*). In particular, we consider natural combinations of epistemic modal logic $\mathsf{S4}$ with the logic of proofs LP. However, this approach is flexible with respect to both the knowledge/belief component for $\Box F$ and the justification component for $t{:}F$, which can be chosen independently. Since there are other known modifications of LP, each capturing its own set of justification properties, there is a variety of systems for epistemic logic with justification.

Formalization of justification significantly expands the expressive power of epistemic logic and provides a new tool for formal studies in epistemology and applications. Here are some epistemological notions which seem to be affected by this new development.

1. The foundational *Gettier problem* of augmenting the tripartite JTB definition of knowledge becomes a formal epistemology issue.

2. The traditional Hintikka-style modal logic approach to knowledge has the well-known defect of *logical omniscience*, caused by an unrealistic stipulation that an agent knows all logical consequences of his/her assumptions. Because of this defect, the usual epistemic modality $\Box F$ should be regarded as "potential knowledge" or "knowability" rather than actual knowledge. Epistemic systems with justification address the issue of logical omniscience in a natural way. A justified knowledge $t:F$ cannot be asserted without presenting an explicit justification t for F, hence justified knowledge is not logically omniscient.

3. Epistemic logic with justifications offers a new approach to *common knowledge*. A new modal operator $J\varphi$ for *justified knowledge* is defined as a forgetful projection of justification assertions $t:\varphi$ in a multi-agent epistemic logic with common justification. It turned out that justified knowledge is a special constructive version of common knowledge and can be used as such in solving specific problems. Justified knowledge is considerably more flexible and in many respects easier than the traditional common knowledge.

Elementarity and incompleteness for predicate modal logics with constant domains

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The results presented here show that incompleteness and elementarity in predicate modal logic are closely related: very often elementary modal logics are incomplete. This contrasts with the propositional case in which the existence of elementary incomplete logic is an open problem [1].

Our modal language $\mathcal{ML}$ contains the unique modality $\Box$, arbitrary predicate symbols (without equality); function symbols or individual constants are not allowed.

Modal formulas are interpreted in Kripke frames with constant domains (W, R, D), where $W, D \neq \emptyset$ and $R \subseteq W \times W$ is transitive.

These frames are characterized with the well-known logic **QK4B** obtained from the classical predicate logic, by adding the axioms schemes:

$$\Box(A \to B) \to (\Box A \to \Box B),$$

$$\forall x \Box A \to \Box \forall x A, \qquad \Box A \to \Box\Box A$$

and the rule $A/\Box A$.

Definition. A modal formula A is called *elementary*, if the class of all frames F validating A is elementary (if they are regarded as classical two-sorted first-order structures).

Definition. For a formula A and $n > 0$ we construct the propositional modal formula A^n as follows. First we replace every occurrence of $\exists x B(x)$ with the disjunction $\bigvee\limits_{1 \le i \le n} B(a_i)$, and then replace every occurrence of every atomic formula $P_j(a_{i_1}, \ldots, a_{i_k})$ with a new proposition letter $q_j^{i_1 \ldots i_k}$.

Definition. The *rank* of a modal formula A (notation: $rk(A)$) is the minimal n such that for all Kripke frames $F = (W, R, D)$

with $|D| \geq n$, we have $F \models A$ iff $F \models A^n$. If such n does not exists, we put $rk(A) := \infty$.

Let U^n be a classical first-order formula (without equality) such that $(W, R, D) \models U^n$ iff $|D| \leq n$.

Theorem. Suppose A is elementary. Then $rk(A) < \infty$ and for all Kripke frames F, $F \models A$ iff $F \models \bigvee_{1 \leq i < n} (A_i \wedge U^i) \vee A^n$, where $n = rk(A)$.

Now let us consider a certain class of modal formulas and describe all elementary formulas in this class. Let S be the set of all formulas of the form

$$M_1 \exists x M_2 P(x) \to N_1 \exists x N_2 P(x),$$

where M_1,M_2,N_1,N_2 are positive modalities such that $M_1 M_2$ and $N_1 N_2$ coincide (*modal reduction principles*).

Theorem. $A \in S$ is non-elementary if and only if $M_1 = N_1 N_3$, where N_3 is a positive modality containing $\Box$, and one of the following conditions holds:

(1) $M_2 = \Diamond^k,\ k > 0$;

(2) M_2 is empty, N_3 ends with $\Diamond$;

(3) M_2 begins with $\Box$, $N_1 = \Box^k$ $(k \geq 0)$, $N_3 = \Box^l$ $(l > 0)$.

Corollary. $A \in S$ is elementary iff $rk(A) < \infty$. Moreover, if A is elementary, then $rk(A) \leq 2$ (and hence A is semantically equivalent to $U^1 \vee A_2$).

Theorem. If $A \in S$ is elementary and $rk(A) = 2$, then the logic **QK4B** $\oplus\, A$ is Kripke incomplete.

The same method is applicable to intermediate logics. In particular, the well-known logic **QH** $\oplus\, (\neg\neg\exists x P(x) \to \exists x \neg\neg P(x))$, which is incomplete [2], is elementary; on the other hand, the logic **QH**$\,\oplus (\forall x \neg\neg P(x) \to \neg\neg\forall x P(x))$, which is complete [3], is non-elementary.

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Japaridze's polymodal logic and its provable and PA-unprovable properties

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Giorgi Japaridze introduced his polymodal provability logic GLP in 1986 and proved its arithmetical completeness w.r.t. a natural provability semantics. GLP is known to be Kripke-incomplete. Further modal logic properties of GLP were investigated by Konstantin Ignatiev who, in particular, established Craig's interpolation property and the fixed point property for GLP. Ignatiev's methods, however, relied on nontrivial assumptions unprovable in Peano arithmetic, such as reflection principles and/or transfinite induction. He even conjectured that such assumptions could actually be necessary. Later, GLP was used (by the speaker) to give a consistency proof for Peano arithmetic a la Gentzen, which made these conjectures more intriguing.

We disprove Ignatiev's conjecture in that we show that a great deal of results on GLP can be obtained by methods formalizable in elementary arithmetic. (Part of this work is done jointly with J. Joosten and M. Vervoort.) However, we also present a modal-logical property of GLP which is unprovable in Peano arithmetic. We also give a novel complete generalized Kripke semantics for GLP.

Reasoning with causal statements: A proto-logic of counterfactuals

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We are here concerned with specifying a logic for reasoning from causal statements. At first sight, a logic of counterfactuals seems a good choice for the following reason. According to the famous counterfactual analysis of causation, an event c causes an event e iff there is a series of events E1,...,En such that: if c had not occurred, E1 would not have occurred; if E1 had not occurred, E2 would not have occurred; ... and if En had not occurred, e would not have occurred. Indeed, this was the motivation for David Lewis to define his logics of counterfactual in the first place.

We discuss why such a logic is not quite right when it comes to formalizing inferences from causal statements. We propose a very weak logic to conform with the cases discussed, suggesting a couple of ways to improve it and stating what the major obstacle to extending it to a full-fledged logic of causation is.

Engineering Modal Logics for Reasoning about Agency, Time, and Obligation

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Two central concepts showing up in many definitions for (software) agents, are autonomy and pro-activity. Autonomy of agents can be described as being able to act in violation with given obligations. Pro-activity can be described as the ability to base decisions about what to do on considerations about the future. So, by definition, or at least by some definitions, reasoning about time and obligation is central to what it means to be an agent. We argue that for the use of temporal deontic logics as knowledge representation languages, the notion of deadline is of crucial importance. We show that there are many possible semantics for deadlines, which is partially due to a variety of possible semantics for the notion of control. The notions of time, obligation and control are closely related, since, if agent's temporal obligations are about things they do not control, they will not have any influence on the decisions they make. In stead of having separate logics for each possible semantics, we study how to define each semantics as a reduction to ATL (alternating time temporal logic). This has many advantages. We can use the logical machinery of ATL (axiomatization, model checking algorithms), to do reasoning. We can check properties of deadline logics by translating and proving them in ATL. This activity of checking the properties of ever changing semantics and reductions can be described as a way of engineering logics; hence the title of the talk.

An algorithmic version of Blok's theorem

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A well-known Blok's theorem [1, Theorem 10.59] states the following:

Let L be a consistent normal modal logic, non-equal to the minimal normal modal logic $\mathbf{K}$. *If L is union-splitting, then the incompleteness degree of L is 1, otherwise the incompleteness degree of L equals continuum.*

Recall that here the incompleteness degree of a logic L is the cardinality of the set of all normal modal logics having the same Kripke frames as L. So we can interpret Blok's theorem as a criterion for coincidence of logics with the same Kripke frames.

The problem of coincidence (or equivalently, non-coincidence) of logics is quite common in logical investigations. For example, consistency of a logic is equivalent to its non-coincidence with the inconsistent logic; derivability of a formula φ in a logic L is the problem of coincidence $L =^? L \oplus \varphi$. The problem of axiomatization of a certain logic, the problem of independence of a given axiom system etc. are also within this area.

In the case when only finite axiomatizations and respectively, finitely axiomatizable logics, are considered (only above $\mathbf{K}$, in this talk), the coincidence problem becomes algorithmic. For example, the problem of derivability in a logic L is exactly the decidability problem. It is also worth noting that from the algorithmic point of view, it is sufficient to consider logics with a single extra axiom above $\mathbf{K}$. For example, we can understand algorithmically the problems of inconsistency and coincidence with $\mathbf{K}$ (for a given formula φ) as follows: $\mathbf{K} \oplus \varphi =^? \mathbf{K} \oplus \bot$ and respectively, $\mathbf{K} \oplus \varphi =^? \mathbf{K} \oplus \top$. Note that the first problem is decidable by Makinson's theorem [1, Theorem 17.2], the second problem is decidable due to the decidability of **K**.

Theorem. *Let L be a consistent normal modal logic nonequal to the minimal normal modal logic* **K**. *Then the problem (with parameter φ)* $\mathbf{K} \oplus \varphi =^{?} L$ *is decidable iff L is union-splitting.*

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Algorithmic problems in semantics of Visser's formal propositional logic

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Joint work with Chagrov A.V.

The logic **FPL** (so-called *fixed point logic*) was introduced by A. Visser in [2] as a rather natural deductive calculus. It has the same language as intuitionistic propositional logic **Int** and it can be embedded into Gödel – Löb provability modal logic **GL** by the well-known translation which embeds **Int** into **S4**: the operator $\Box$ is added to (only!) atomic formulas and implications. The semantics of **FPL** introduced in [2] is similar to the (finite) Kripke semantics for **Int**; frames are now finite strict orders (so unlike the case of **Int**, irreflexivity is postulated), and all other details of the definitions are the same. Thus frames of the logic **FPL** (as structures) are exactly the finite frames of **GL**; the class of these frames is denoted by $\mathcal{FSO}$.

In the talk algorithmic problems for $\mathcal{FSO}$ are considered, analogous to those studied in [1] for the case of the finite Kripke semantics for **GL**. Although in both cases the problems look similar, their solutions are different. In particular, in the semantics for **FPL** there is no difference between the global and the local truth, but for **Int** these notions are non-equivalent. Another feature is that "negative" algorithmic results for $\mathcal{FPL}$ probably cannot be obtained for formulas with small number of variables. The talk discusses these similarities/non-similarities in detail. Some results on finite-variable fragments of modal logics from [1] are also strengthened. Here are two examples.

Theorem. *The logical consequence relation "a formula ψ is valid in all $\mathcal{FSO}$-frames validating a formula φ" is undecidable in the case* **GL**, *and in the case* **FPL**. *It is sufficient to* *consider one-variable formulas in the first case and two-variable formulas in the second case.*

Theorem. *First-order definability of formulas over the class $\mathcal{FSO}$ is undecidable in the case of* **GL***, and in the case* **FPL***. It is sufficient to consider one-variable formulas in the first case and two-variable formulas in the second case.*

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Towards algorithmic correspondence and completeness in modal logic

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Joint work with Willem Conradie and Dimiter Vakarelov

Correspondence and completeness are key topics of the model theory of modal logic. A celebrated general result relating these two concepts for a large syntactic class of modal formulae is Sahlqvist's theorem. Sahlqvist's approach to proving first-order definability and canonicity of modal formulae was paralleled and further developed by van Benthem into the substitution method.

Proving first-order definability of modal formulae amounts to elimination of second-order quantifiers. Two algorithms have been developed and implemented for elimination of predicate quantifiers in second-order logic: SCAN, based on a constraint resolution procedure, and DLS, based on a logical equivalence established by Ackermann.

In this talk I will introduce a new algorithm, SQEMA, for computing first-order equivalents, and at the same time proving canonicity, of modal formulae. Like DLS, it uses (a modal version of) Ackermann's lemma, but unlike both SCAN and DLS it works directly on modal formulae and thus avoids introduction of Skolem functions and the subsequent problem of unskolemization. If successful, the algorithm produces a locally equivalent pure formula in a temporal (reversive) extension with nominals of the input language.

In return for being specialized, SQEMA is more transparent and flexible, easier to use and extend, less dependent on the syntactic shape of the formulae, and apparently has a wider scope of applicability on modal formulae than either of the others.

In this talk I will present the core algorithm and illustrate it with some examples. I will then discuss prove its correctness and show that it succeed not only on all Sahlqvist formulae, but also on the larger class of inductive formulae, introduced by Goranko and Vakarelov.

Since all formulae on which SQEMA succeeds are provably canonical, we have thus introduced a purely algorithmic approach to proving completeness via canonicity in modal logic and, in particular, established what we believe to be the most general completeness result in modal logic so far.

A decidable modal logic that is undecidable on finite frames

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In the fields of Computer Science and Artificial Intelligence there is a natural question: how different can be properties of finite and infinite structures? A well-known result of this kind is Trahtenbrot's theorem stating that the logic of all finite models of classical first-order logic is not recursively enumerable. But many problems remain open in this area.

In particular, consider the following property. A logic (or a theory) L is called *finitely decidable* if the logic (resp., theory) of all its finite models is decidable; otherwise L is called *finitely undecidable.*

No examples of decidable logics and theories that are finitely undecidable have been known until recently. Apparently, the first example of a first-order (equational) decidable theory that is finitely undecidable, was described by J. Jeong [1].

In the talk we give an example of a normal modal logic with the same properties:

Theorem. *In* NExt**GL** *there exists a decidable logic that is finitely undecidable.*

The proof of this theorem essentially uses Zakharyaschev's canonical formulas (see [2]).

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Programming modal tableaux systems

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We define formal tools for the definition of tableaux systems for modal logics. First we present a high-level declarative programming language based on graph rewriting rules together with control constructs for defining strategies of application of these rules. This allows us to make explicit and rigorous the usually implicit and/or operational notions of rule application and strategy. Secondly, this allows us to state general termination theorems for two classes of strategies. These results cover almost all basic modal logics such as K, T, S4, KB4, and even LTL and PDL. Our framework provides the theoretical basis for our generic tableau theorem prover LoTREC.

Set theory with modality

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Let ECT be the assumption that all ”rules” are recursive. The result of M.Beeson and A. Scedrov [1] is IZF+ECT $\nvdash$ KSL (the theorem about a continuity of effective operators: for HA this theorem was proved by Ceitin and by Kreisel, Lacombe and Shoenfield). The method of Beeson and Scedrov is fp-realizability for HA, which ”lifts” to set theory level. Here the authors used intermediate set theory with modality. This set theory was not further investigated and the aim of my report is to draw the attention to the theme: high formal systems with modal underlying logic.

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Formulas, corresponding to diagrams

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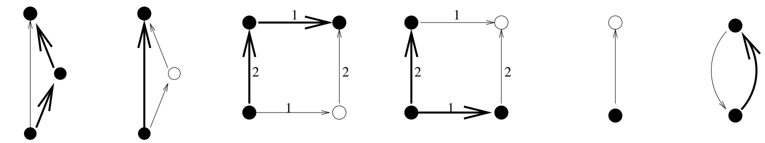
Many first-order properties are illustrated with pictures, see table.



transitivity density commutativity Church–Rosser's property seriality commutativity

Our goal is to formalise the notion of a diagram, then assign a first-order formula to a diagram, and investigate properties of these formulas in the modal context.

A *diagram* is a quadruple $D =< W, B, P, r >$, where W is a finite set of worlds, B and P are binary relations on W, and $r \in W$. Relation B corresponds to black, or bold, arrows, and P corresponds to dashed, or white, arrows. We call a point $x \in W$ *black* if there is an non-oriented path from r to x consisting of black arrows, all other points are called *white*. Let $W_B = \{x|x \text{ is black }\} = \{r = x_0, x_1, \ldots, x_n\}$. The white points are $y_1, \ldots, y_m$.

A diagram is *correct* if the following conditions hold:
(1) for any black point x there is unique oriented path joining r and x (i.e. (W, B) is a tree with root r);
(2) for any white point y there is a black point x such that there is a path, consisting of white arrows joining y and x;
(3) (minimality) if we delete any white arrow, then the new first-order property is non-equivalent to the old one.

Henceforth we consider only correct diagrams. To each point $x \in W$ we assign a variable v_x. To every diagram D we assign a first-order formula E_D with a single free variable v_r in the language with a single binary relation R:

$$E_D = \forall v_{x_1} \ldots \forall v_{x_n} \left(\left(\bigwedge_{xBy} v_x R v_y \right) \to \exists v_{y_1} \ldots \exists v_{y_m} \left(\bigwedge_{xPy} v_x R v_y \right) \right).$$

In the above conjunctions the variables x and y run over W. Note that the formula E_D is *guarded* (cf. [1]).

Now consider the basic modal language.

Theorem 1. The first-order condition with a correct diagram is localy definable in the basic modal language iff the corresponding diagram does not have non-oriented cycles consisting of white arrows and white points. Moreover, if the diagram is definable, it is definable by a canonical formula.

Now consider the case when the root r is the only black point of the diagram (so-called *strongly existential formulas*). Let $\mathcal{C}$ be the class of all pairs (F, r) consisting of a rooted frame and its root, such that $F \models E_D(r)$. $L(\mathcal{C})$ denotes the set of modal formulas ϕ such that $F, r \models \phi$ for all $(F, r) \in \mathcal{C}$. Note that $L(\mathcal{C})$ is not a normal modal logic since it is not closed under Generalization. Nevertheless we can propose an infinite axiomatisation for $L(\mathcal{C})$, even for the case when the diagram is not modally definable.

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Modal products and dynamic topological logics

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Joint work David Gabelaia, Boris Konev, Agi Kurucz, Frank Wolter and Michael Zakharyaschev

In this paper, we present our recent results on products of 'transitive' modal logics with constant and expanding domains. We use these results - together with the developed techniques - to investigate the computational behaviour of dynamic topological logics introduced in 1997 by Artemov, Davoren, Kremer, Mints, and Nerode.

Our first result - a solution to a well known open problem - shows that the product of two (Kripke complete) 'transitive' modal logics with frames of arbitrary finite or infinite depth can never be decidable, with the complexity ranging from r.e. to non-r.e., and even worse. As a consequence, we give the first known examples of Kripke incomplete commutators of Kripke complete logics.

Then we show that - unlike full products of 'transitive' modal logics - their 'expanding domain' relativisations can be decidable, though not in primitive recursive time. This is true, in particular, of product logics determined by product frames one component of which is a finite linear order or a finite transitive tree and the other is a transitive tree or a partial (quasi- or linear) order. However, if we allow the first component to be isomorphic to (N,¡) then the logic becomes undecidable, yet recursively enumerable whenever the second component contains only finite frames. The proofs are based on Kruskal's tree theorem and a reduction of various reachability problems for lossy channel systems.

Finally, we apply the results above in the framework of dynamic topological logics (DTLs). Roughly, we obtain the following landscape. DTLs over various natural dynamical topological systems with homeomorphisms are usually not recursively enumerable, even if we allow only finitely many iterations. DTLs over models with continuous mappings and finitely many iterations become decidable, but not in primitive recursive time. DTLs over arbitrary Aleksandrov spaces and continuous mappings are again undecidable (though possibly axiomatisable).

We also briefly discuss the impact of a third dimension by showing that first-order intuitionistic logic with two variables is undecidable.

Extensions of multidimensional successor logics

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Modal logic **SL** (*Tomorrow*, or *Successor Logic*) was introduced by E.Lemmon and D.Scott (1965) and K.Segerberg (1967); all extensions of this logic were described by A. A. Muchnik [1]. Muchnik's theorem was generalized by Segerberg for all extensions of $\mathbf{KAlt_1}$ (the logic of partial functions) [4].

Later polymodal versions of **SL** were introduced and systematic study of their extensions (logics with functional modalities) was started. However this class is too large: it contains undecidable modal logics of rather simple form; moreover, as shown in [3, Section 9.4] a reasonable classification of this class cannot be obtained.

However an interesting subclass of functional polymodal logics are those, in which all modalities commute. These logics are extensions of multidimensional successor logics, i.e. of products $\mathbf{SL}^n$ described in [2, Section 14]:

$$\mathbf{SL}^n := \underbrace{[\mathbf{SL}, \mathbf{SL}, \dots, \mathbf{SL}]}_{n}.$$

For this subclass several rather strong results can be proved:

Theorem.

(1) All extensions of $\mathbf{SL}^n$ are finitely axiomatisable.

(2) All extensions of $\mathbf{SL}^n$ are decidable and, moreover, the deducibility problem for them is decidable.

(3) All extensions of $\mathbf{SL}^n$ have the f.m.p.

(4) If $\mathbf{\Lambda} \supseteq \mathbf{SL}^n$ and $\mathbf{\Lambda}$ is a logic of a single cone, then for any Kripke-complete polymodal logic $\mathbf{L}$, $\mathbf{\Lambda}$ and $\mathbf{L}$ are product-matching.

(5) for $\mathbf{L} = \mathbf{K}_m$ the converse of (4) holds , i.e. if a consistent $\mathbf{\Lambda} \supseteq \mathbf{SL}^n$ and K_m are product-matching, then $\mathbf{\Lambda}$ is a logic of a single cone.

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Explicit modal logics of single-conclusion proof systems

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Joint work with Sergei Artemov and Nikolai Krupski

Explicit modal logics, also known as logics of proofs, provide high level symbolic description of proofs as objects in a classical logic environment. Such logics may be regarded as user friendly interfaces for reasoning about proofs. In this talk we will deal with explicit modal logics corresponding to the most common type of proof systems, single-conclusion proof predicates.

Two approaches are considered. The first one is based on the unification technique. It leads to the extension of the language by reference constructions of the form "*the formula that is proven by t*" combined with pattern matching. The resulting logic FLP_{ext} is decidable and provides a complete admissibility test for arithmetical inference rules specified by schemes in the language of FLP_{ext}.

The second approach involves rigid typing and leads to a system $\mathsf{RCL}_{\rightarrow}$ that is a reflexive extension of Curry's combinatory logic $\mathsf{CL}_{\rightarrow}$. In this system for each proof term t a formula F, for which the formula "*t proves F* " is well-formed, is unique. Well-formness of a formula in $\mathsf{RCL}_{\rightarrow}$ can be tested in polynomial time. The derivability problem for $\mathsf{RCL}_{\rightarrow}$ is *PSPACE*-complete.

Topological modal logic with the difference modality

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The basic language used in topological modal logic is well known and dates back to McKinsey – Tarski [1]. It has a single modal connective $\Box$ interpreted in topological spaces as the interior operator. But this language does not have much expressive power. So we add difference modality $[\neq]$ and consider the bimodal language $\mathcal{ML}(\Box, [\neq])$, which happens to be more expressive. The semantics for $[\neq]$ is standard: a formula $[\neq]A$ is true at point x iff A is true at all points y such that $y \neq x$. The universal modality is expressible in an obvious way: $[\forall]A = [\neq]A \wedge A$.

Our basic logic **S4D** is the fusion of **S4** for $\Box$ and **DL** (Difference Logic) for $[\neq]$ plus $[\forall]p \to \Box p$ (see [2]).

We also consider the following extra axioms:

$$
\begin{array}{cl}
(AT_1) & [\neq]p \to [\neq]\Box p \\
(ADs) & [\neq]p \to \Diamond p \\
(AC) & [\forall](\Box p \vee \Box\neg p) \to [\forall]p \vee [\forall]\neg p \\
(AE_1) & [\neq]p \wedge \neg p \wedge \Box(p \to \Box q \vee \Box\neg q) \to \Box(p \to q) \vee \Box(p \to \neg q)
\end{array}
$$

Lemma 1. Let X be a topological space. Then
(1) $X \models AT_1$ iff X is a T_1-space;
(2) $X \models DS$ iff X is dense-in-itself;
(3) $X \models AC$ iff X is connected. [3]

Definition 2. A topological space is called *locally connected* if every neighborhood of any point contains a connected sub-neighborhood. A locally connected T_1-space is called *locally 1-connected* if the complement of a point in every its connected open subspace is also connected.
Lemma 3. Let X be a locally connected T_1-space. Then
$X \models AE_1$ iff X is locally 1-connected.

Consider the following modal logics:
$\mathbf{S4DT_1S} := \mathbf{S4D} + AT_1 + ADs$.

$\mathbf{S4DEC} := \mathbf{S4DT_1S} + AC + AE_1$.

Theorem 4. $\mathbf{S4DT_1S}$ and **S4DEC** have the FMP.

Theorem 5. (1) **S4D** is the logic of all topological spaces.
(2) $\mathbf{S4DT_1S}$ is the logic of Cantor's space.
(3) **S4DEC** is the logic of $\mathbb{R}^n$ for any $n \geq 2$.

Open questions: (1) find the logic of $\mathbb{R}$;
(2) Is $\mathbf{S4DT_1} = \mathbf{S4D} + \mathbf{AT_1}$ complete in topological semantics?

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The Modal Logic of Agency

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Although there had already been done quite some work on the logic of action in philosophical logic, the interest in cognitive science and artificial intelligence for intelligent or cognitive agents and their behaviour that has developed over the last 15 years, has increased the logical investigation of agency even further. In particular, it appeared that modal logic could be fruitfully employed to describe and specify the behaviour of cognitive agents using modal operators for the mental attitudes of those agents, such as knowledge, belief, desire, intention, goal, commitment, etc.

The seminal works in this area are those of Cohen & Levesque and Rao & Georgeff. Both try to formalize in modal logic the philosophical theory of Bratman about the role of intentions in human decision-making and planning. In order to describe the behaviour of agents Cohen & Levesque and Rao & Georgeff use a temporal logic framework, on top of which they put modal operators for the agent's mental attitudes like belief, desire and intention (BDI). (The former authors employ a linear-time setting whereas the latter use a branching-time setting.) We ourselves have proposed a logic of action rather than a logic of time to describe / specify agents, viz. dynamic logic, on which we put several other modal operators dealing with the mental attitudes. Recently I've been able to extend this framework to one for describing cognitive aspects of agents beyond rationality (BDI), viz. emotional behaviour.

A game semantics for intuitionistic propositional logic

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For every intuitionistic propositional formula F we construct a game G_F between Proponent (**P**) and Opponent (**O**) as described below.

Game field. A game field is the syntax tree of F. Identical subtrees can be merged (turning the tree into a dag); the game outcome is not affected.

Stones. Both players have stones of their color (say, white for **P**, black for **O**). Each vertex has a room for one stone to be placed on it. In order to make a move, a player places a stone of his color onto a vacant vertex. Once placed, a stone remains in its vertex forever.

Publishing. Another kind of move is available to **O**: he may "publish" a variable. This means simply saying, for instance, "I publish p". Each variable may be published only once.

Positions. A position is a tuple $(\mathrm{PV}, \mathrm{PS}, \mathrm{OS})$, where PV is the set of published variables, PS is the set of **P**'s stones (a set of vertices carrying a **P**'s stone), and OS is the set of **O**'s stones. PS and OS may not intersect. All three sets grow larger as the position develops.

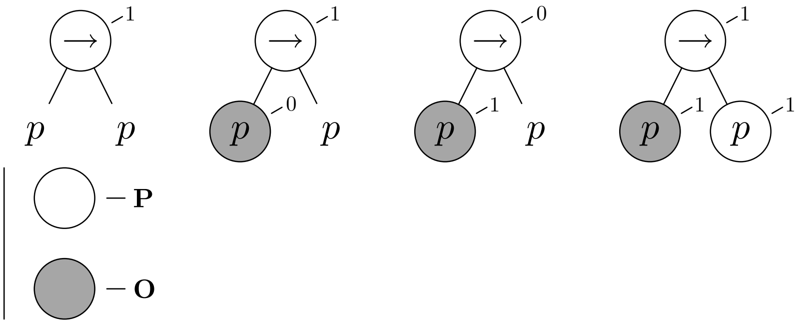
Starting. The starting position is $(\emptyset, \{\mathrm{root}\}, \emptyset)$, that is, **P**'s only stone is in the root.

States. In any position, each vertex has a state, which can be 0 or 1. States are needed only to determine whose turn it is. A vacant vertex is 0. An occupied variable is 1 iff the variable is published. An occupied non-variable behaves according to its boolean function.

Turn. If **P** has a stone in state 0 but all **O**'s stones are in state 1, then it is **P**'s turn to move. Otherwise, it is **O**'s turn.

Who wins. Whoever has a turn but cannot make a move, loses. The other player wins.

Example. This simple example shows playing $p \to p$. The small digits mean states. States of vacant vertices are not shown (they are always 0).



Between the second and the third pictures, **O** publishes the variable p. The fourth picture shows **P** winning: it's **O**'s turn, but **O** cannot make a move.

This game is a precise semantics for IPL:
Theorem. A propositional formula F is derivable in IPL iff **P** has a winning strategy in G_F.

De Jongh's Theorem for equality theories

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Definition. Let T be a consistent intuitionistic first-order theory. *De Jongh's theorem for T* is the following property:

For any propositional formula $A(p_1, \ldots, p_n)$,
Int $\vdash A(p_1, \ldots, p_n)$ iff for all T-sentences $B_1, \ldots, B_n, T \vdash A(B_1, \ldots, B_n)$.

Similarly, *De Jongh's uniform theorem* is the following property:

There is a sequence of T-sentences $B_1, \ldots, B_n, \ldots$ such that for any propositional formula $A(p_1, \ldots, p_n)$,
Int $\vdash A(p_1, \ldots, p_n)$ iff for all T-sentences $B_1, \ldots, B_n, T \vdash A(B_1, \ldots, B_n)$.

These properties were first established for the intuitionistic arithmetic **HA** ([1]) and most work in this area concerns extensions of **HA**. A survey of these results can be found in [3].

Our results are about De Jongh's theorem for theories of equality as defined below:

Definition. A consistent intuitionistic first-order theory with equality without predicate symbols, function symbols, or constants is called *a theory of equality.*

In particular, *the minimal theory of equality* **IntEq** has no non-logical axioms, *the theory of normal equality* **IntNormEq** has the axiom $\forall x \forall y\ (\neg\neg x = y \to x = y)$, and *the theory of decidable equality* **IntDecEq** has the axiom $\forall x \forall y\ (x = y \vee \neg x = y)$.

Theorem 1. De Jongh's theorem holds for all theories between **IntEq** and **IntDecEq**.

This is proven essentially following Smoriński's proof from [2] for arithmetic. There is also a negative result:

Theorem 2. De Jongh's uniform theorem does not hold for the theory of normal equality.

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Evidence-based knowledge for S5

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The Logic of Evidence-based Knowledge was proposed by S.N. Artemov in [1]. This is a multi-agent logic of knowledge enriched by evidence assertions of the form $t{:}\varphi$ where t is an *evidence term*. The knowledge of an agent is axiomatized by a modal logic $\mathbf{L}$ (where $\mathbf{L}$ may be $\mathbf{T}$, or $\mathbf{S4}$, or $\mathbf{S5}$). The evidence-based knowledge operator is described by the logic of proofs $\mathbf{LP}$ (see [2]). The resulting systems $\mathbf{L}_n\mathbf{LP}$ have a Kripke-style semantics, for which the corresponding completeness theorems are established. It is also known that the forgetful projection (replacing all evidences by a uniform modality J) transforms a logic $\mathbf{L} = \mathbf{T}$ or $\mathbf{S4}$ into $\mathbf{L}_n\mathbf{S4}$.

We introduce the multi-agent logics of evidence-based knowledge $\mathbf{S4}_n\mathbf{LP}(\mathbf{S5})$ and $\mathbf{S5}_n\mathbf{LP}(\mathbf{S5})$, in which the evidence component corresponds to an $\mathbf{S5}$-modality. The language of $\mathbf{S4}_n\mathbf{LP}$ is extended by the new unary operation "?" (*negative evidence checker*).

The axioms and rules of $\mathbf{S4}_n\mathbf{LP}(\mathbf{S5})$ are those of $\mathbf{S4}_n\mathbf{LP}$ plus the new axiom:

$$\neg\,(t{:}\varphi) \rightarrow ?t{:}(\neg\, t{:}\varphi).$$

For this logic we define Kripke-style models and prove the completeness theorem. A *model* is a structure $\mathcal{M} = (W, R_1, \ldots, R_n, R, \mathcal{E}, \Vdash)$, where $W \neq \emptyset$ is a set of states, R_i, R are reflexive and transitive accessibility relations on W. An *evidence* function $\mathcal{E}$ is a mapping from states and evidence terms to sets of formulas. It is monotonic (uRv implies $\mathcal{E}(u,t) \subseteq \mathcal{E}(v,t)$) and respects operations on evidence terms. $\Vdash$ is a satisfiability relation between states and sentence variables. Satisfiability of formulas is defined in a usual way with the only new clause: $u \Vdash [\![t]\!]\varphi$ iff $\varphi \in \mathcal{E}(u,t)$. We require that $\varphi \in \mathcal{E}(u,t)$ implies $v \Vdash \varphi$ for all $v \in W$ with uRv.

Theorem. $\mathbf{S4}_n\mathbf{LP}(\mathbf{S5}) \vdash \varphi$ iff φ holds in all $\mathbf{S4}_n\mathbf{LP}(\mathbf{S5})$-models.

In the case of $\mathbf{S5}_n\mathbf{LP}(\mathbf{S5})$ the knowledge of every agent is described by the modal logic $\mathbf{S5}$ and the evidence part is the same as in $\mathbf{S4}_n\mathbf{LP}(\mathbf{S5})$. Accessibility relations of corresponding Kripke-style models are reflexive, transitive and symmetric. A similar soundness and completeness theorem holds for $\mathbf{S5}_n\mathbf{LP}(\mathbf{S5})$.

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Almost periodic sequences

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Definition. A binary sequence u is called *almost periodic* if for any subword a there is a number n such that any subword b of u of length n contains the subword a.

Theorem. For any number α there exist a number n and an almost periodic sequence such that every its subword of length $m \geq n$ has Kolmogorov complexity not less than $m\alpha$.

Complexity of the two-variable fragment of intuitionistic propositional logic

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It is well known that for many standard propositional logics the decision problem is PSPACE-complete. Note that proofs of PSPACE-completeness of non-classical logics usually involve infinitely many variables. However as mentioned in [2], there is a certain interest in studying complexity of finite-variable fragments of **Int** and other non-classical logics — as soon as in most applications deal with formulas built from a finite set of variables.

For a propositional logic L and a natural number n, $L(n)$ denotes the n-variable fragment of L. It is known [1, 3] that the decision problem is PSPACE-complete for $\mathbf{K}(0)$, $\mathbf{K4}(0)$, $\mathbf{S4}(1)$, $\mathbf{T}(1)$, $\mathbf{GL}(1)$, $\mathbf{Grz}(1)$ and is in P for $\mathbf{Int}(1)$ (cf. [4]). So far the complexity of $\mathbf{Int}(2)$ has been an open problem (see Problem 18.4 in [2]).

Theorem. *Let L be a logic such that $\mathbf{Int} \subseteq L \subseteq \mathbf{Int} + \neg p \vee \neg\neg p$. Then the decision problem for $L(2)$ is* PSPACE*-hard.*

As a corollary we obtain that in the language with two variables the following logics are PSPACE-hard: (1) all intermediate logics axiomatized by one-variable formulas (except for **Cl**), (2) all intermediate logics axiomatized by implication free formulas (except for **Cl**), (3) Kreisel–Putnam's logic **KP**, (4) Medvedev's logic **ML**, and others.

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RCC-relations and relativistic time

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Joint work with Valentin Shehtman

We discuss the relationship between modal logic of regions and temporal logic of relativity. Informally, the main idea is that regions in n-dimensional space can be considered as points in $(n+1)$-dimensional spacetime. This correspondence allows us to regard relations of Region Connection Calculus (RCC) as relativistic relations (and vice versa).

An outline of completeness results in relativistic modal logic can be found in [2]. For the described logics the finite axiom systems were presented; moreover, they have the finite model property and are PSPACE-complete. Due to the proposed correspondence, these results can be interpreted for logics of various region structures in $\mathbb{R}^n$ (e.g. balls, rectangles, connected or convex regular sets, and others).

Conversely, results on logic of regions can be reformulated for relativistic logics. For example, there is a number of "negative" results for logics of regions (e.g. [1]): rather expressive systems turn out to be undecidable (or even not recursively enumerable). The proposed method provides the same results for logic of relevant relativistic structures.

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Spatial modal logics and information types

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In this talk we consider three main approaches to spatial modal logics: point-based geometrical approach, topological approach, region-based approach. For each of these types of modal logics we discuss main results and open problems.

We also describe a link between region-based modal logics and "informational semantics" of intuitionistic propositional logic. This semantics was proposed by Ju.T. Medvedev [1] as a possible development of A.N. Kolmogorov's (1933) viewpoint on intuitionistic logic as a logic of problems. The original Medvedev's definition regarding "information" as a non-empty set of integers gives a sound, but incomplete semantics. But completeness can be restored if "information" is a certain kind of region in space (e.g., a disk on a plane, or an interval on a line) [2]. This also leads us to a spatial interpretation of extra modal connectives in intuitionistic logic, like difference or universal modality.

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Multisource algorithmic information theory

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Informally speaking, Kolmogorov complexity of a string x is a number of bits needed to transmit x through a communication channel (from source to destination). One can consider a more elaborated network with several sources and several destinations.

Formally, let (V, E) be a directed acyclic graph. Assume that for some vertices $v \in V$ an *incoming* string $i(v)$ is fixed, for some vertices $v \in V$ an *outgoing* string $o(v)$ is fixed, and for each edge $e \in E$ the *capacity* $c(e)$, which is a non-negative integer or $+\infty$, is given. Then we say that an *information transmission request* is defined.

We say that an information transmission request is *c-feasible* for some $c > 0$ if there exists a function t that maps each edge e into a string $t(e)$ (a string "transmitted along e") of length at most $c(e)$ such that for each vertex v the conditional complexity

$$\mathrm{K}\,(\text{outgoing information at } v \mid \text{incoming information at } v)$$

does not exceed c. By outgoing information we mean a tuple of strings that contains all $t(e)$ for all outgoing edges e and the string $o(v)$ if it is defined for v. Similarly, incoming information is a tuple of strings $t(e)$ for all incoming edges plus $i(v)$ (if defined).

We consider the following class of problems: for a given graph find the necessary or sufficient conditions for the information transmission request to be c-feasible in terms of conditional complexity of incoming and outgoing strings and capacities. As usual, we are interested in asymptotic results where $c = O(\log n)$ and all strings have length at most n (as $n \to \infty$). It turns out that

• many notions and results in algorithmic information theory can be reformulated in this language (examples: conditional information, common information, Muchnik theorem on conditional codes, Bennett – Gacs – Li – Vitanyi – Zurek theorem);

• there is a natural necessary condition of Ford – Falkerson type (information flow through any cut should not exceed its total capacity) that turns out to be sufficient for some graphs (including the graph corresponding to Muchnik theorem and all graphs with only one incoming string and many outgoing strings identical to the incoming one);

• in general this condition is not sufficient (Muchnik – Romashchenko results on common information, M. Vyugin – Muchnik – Ustinov results on irreducible programs, minimal sufficient statistics).

However, it is still unclear, what makes graphs different in this respect and how to decide (looking at the graph) whether the information flow condition is sufficient for this graph.

On non-axiomatizability of superintuitionistic predicate logics of dually well-founded Kripke frames

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We consider superintuitionistic predicate logics (without equality and function symbols), i.e., extensions of intuitionistic predicate logic **QH** closed under Modus Ponens, Universalization and (predicate) Substitution. For these logics we use the standard predicate Kripke semantics. For a class Y of posets let $\mathbf{L}(Y)$ (or $\mathbf{L}^c(Y)$) be the predicate logic characterized by the class of all Kripke frames (or, respectively, all Kripke frames with constant domains) with the structures of possible worlds from Y.

Let WF^* be the class of all dually well-founded posets, i.e., posets without infinite ascending chains. Let P_n be the class of all posets of height $\leq n$ (for $n \in \omega,\ n > 0$) and let $P_\infty = \bigcup_n P_n$ be the class of all posets of finite height. Clearly, $P_\infty \subset WF^*$.

Theorem. Let Y be a class of rooted posets, $Y \subseteq WF^*$.
(1) If $Y \not\subseteq P_\infty$, then the logics $\mathbf{L}(Y)$ and $\mathbf{L}^c(Y)$ are Π^1_1-hard.
(2) If $\forall n\,(Y \not\subseteq P_n)$, then the logics $\mathbf{L}(Y)$ and $\mathbf{L}^c(Y)$ are not recursively axiomatizable.

Corollary. (1) The logics $\mathbf{L}(WF^*)$ and $\mathbf{L}^c(WF^*)$ are Π^1_1-hard.
(2) The logics $\mathbf{L}(P_\infty)$ and $\mathbf{L}^c(P_\infty)$ are not RE.

Note that the logics $\mathbf{L}(P_n)$ and $\mathbf{L}^c(P_n)$ are finitely (and thus, recursively) axiomatizable (Ono [1] and Yokota [3]). Hence the logics $\mathbf{L}(P_\infty) = \bigcap_n \mathbf{L}(P_n)$ and $\mathbf{L}^c(P_\infty) = \bigcap_n \mathbf{L}^c(P_n)$ are Π^0_2-arithmetical.

By the way, note that the logic of the class of all well-founded Kripke frames (i.e., without infinite descending chains) obviously equals **QH**.

The proof of Theorem uses a modification of the method developed for the proof of results from [2].

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Definability over the class of all partitions

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Joint work with Ph. Balbiani

Study of correspondence between modal logic and first-order logic, when they are interpreted in relational structures, i.e. Kripke frames of the form $\mathcal{F} = (W, \mathcal{R})$, has been extensive since the 1960s. One of the main topics in this investigation is correlation between expressive power of modal and classical first-order languages. So the major work in correspondence theory has been focused on modal definability of first-order sentences and first-order definability of modal formulas. The most interesting is a series of results on modal and first-order definability proved by Chagrova in the 1990s. In particular, Chagrova's work implies that it is undecidable whether an arbitrary first-order sentence has a modal correspondent within the class of all relational structures. The other way round, it is undecidable whether an arbitrary modal formula has a first-order correspondent within the class of all relational structures.

With these basic results mentioned, we can now confine ourselves to partitions, i.e. Kripke frames $\mathcal{F} = (W, \mathcal{R})$, where $\mathcal{R}$ is reflexive, symmetric, and transitive. They are considered as frames for three modal languages: the standard one, the modal language with the universal modality and so-called 'δ-language' (a fragment of the modal language with the universal modality). For each of these languages we give an appropriate model characterization of first-order formulas having a modal correspondent. Using these characterizations we prove algorithmic decidability of modal definability over the class of partitions and determine its complexity. For all three modal languages every modal formula has a first-order correspondent over the class of all partitions.

Operations on proofs and labels

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Logic of proofs $\mathcal{LP}$ introduced by S. Artemov in 1995 describes properties of the proof predicate *"t is a proof of F"* in the propositional language extended by atoms of the form $t : F$. Proof are represented by terms constructed by three elementary recursive operations on proofs.

We consider the language of $\mathcal{LP}$ as a convenient framework, in which operations on proofs can be described and studied. To make the language more expressive, we introduce the additional *storage predicate* with the intended interpretation *"x is a label for F"*. It can play the role of syntax encoding, so it is useful for specification of operations that require formula arguments.

We study operations on proofs and labels that can be specified in the extended language. We give a formal definition for a specification of an operation on proofs and labels. This definition is purely semantical. The syntactical criteria whether a formula is a specification is proven.

For an arbitrary finite set of operations $\mathcal{F}$ the logic $\mathcal{LPS}(\mathcal{F})$ is defined. We provide $\mathcal{LPS}(\mathcal{F})$ with symbolic and arithmetical semantics. The main result is the uniform completeness theorem for a wide class of these logics with respect to both types of semantics. As a corollary we obtain decidability of these logics and complexity bounds.

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On Kripke-style semantics for the provability logic with quantifiers on proofs

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We consider first order extensions of the logic of proofs, in which we do not change the set of atomic formulas, but allow for quantification over proof variables. In this language the modal provability operator $\Box A$ is expressed by the formula $\Box A \rightleftharpoons \exists x(x : A)$. One can also state the operational properties of provability by the corresponding $\forall\exists$-sentences, e.g.

$$(\cdot) \quad \forall xy \exists z(x : (A \to B) \to (y : A \to z : B)),$$
$$(!) \quad \forall x \exists y(x : A \to y : x : A).$$

We can also formulate further valid principles such as

negative introspection $\quad \forall x \exists y(\neg(x : A) \to y : \neg(x : A)),$
reflection rule $\quad \forall x \exists y(x : \Box A \to y : A)$

Some non-axiomatizability results for different versions of provability logic with quantifiers on proofs are found in [1]. However, there is a hope that the logic corresponding to the standard Gödel proof predicate is decidable. Some partial positive results in this direction are presented in [2, 3]. The solution of this problem is closely related to the task of finding adequate Kripke-style semantics for this kind of logics.

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