Report on the work of A. Skopenkov in 2006
in frame of the Pierre Deligne Stipendium

The following papers were posed at the internet, submitted, accepted or published in 2006.

[1] A. Skopenkov, Embedding and knotting of manifolds in Euclidean spaces, London Math. Soc. Lect. Notes, to appear. http://arxiv.org/ math.GT/0604045

[2] A. Skopenkov, Classification of smooth embeddings of 3-manifolds in the 6-space, submitted. http://arxiv.org/ math.GT/0603429

[3] M. Kreck and A. Skopenkov, Classification of smooth embeddings of 4-manifolds in the 7-space, submitted. http://arxiv.org/ math.GT/0512594 (in 2006 a new version was sent).

[4] A. Skopenkov, Classification of embeddings below the metastable dimension, preprint, http://arxiv.org/ math.GT/0607422

[5] A. Skopenkov, A characterization of submanifolds by a homogeneity condition, submitted. http://arxiv.org/ math.GT/0606470

[6] D. Goncalves and A. Skopenkov, Embeddings of homology equivalent manifolds with boundary, Topol. Appl., 153:12 (2006) 2026-2034

[7] M. Cencelj, D. Repovs and A. Skopenkov, Codimension two PL embeddings of spheres with nonstandard regular neighborhoods, Chinese Annals of Mathematics, Series B, to appear. http://arxiv.org/ math.GT/0608653.

[8] A. Kaibkhanov and A. Skopenkov, Examples of transcendent numbers (in Russian), Mat. Prosveschenie, 10 (2006), 176–184. http://www.mccme.ru/ free-books/ matprosb.html

[9] A. Skopenkov, Olympiads and mathematics (in Russian), Mat. Prosveschenie, 10 (2006), 57–63. http://www.mccme.ru/ free-books/ matprosb.html

The paper [1] is a survey paper. The main results are in papers [2], [3] and [4]. Their abstracts are available in the internet.

In [2] I obtain a complete concrete classification of smooth isotopy classes of smooth embeddings of closed orientable 3-manifolds into the 6-dimensional Euclidean space. This classification amounts to description of action of the group of smooth embeddings $S^3 \to S^6$ on the above isotopy classes of embeddings. Our results imply that such an action is effective, but can be non-free for some 3-manifolds, which is very surprising. The proof is based on modification of sugery theory developed by Kreck and on the Boechat-Haefliger smoothing formula.

In [3] we obtained results on action of smooth embeddings $S^4 \to S^7$ on the set of isotopy classes of embeddings of a closed 4-manifold of S^7. We show that such an action could be trivial for some 4-manifolds (e.g. CP^2), which is very surprising. The proof is based on modification of sugery theory developed by Kreck and on the Boechat-Haefliger theorem. In the 2006 version there is a stronger version of the Effectiveness Theorem and a clearer exposition of its proof involving important results in the theory of smoothing PL embeddings and isotopies.

In [4] the deleted product method is enriched to obtain new results on isotopy classification of embeddings of manifolds. Specifically, new results appeared in the PL category for embeddings of d-connected n-dimensional closed manifolds into R^m and $2m = 3n + 2 - d$. This is in some sense the first non-trivial case as explained in the introduction. An invariant is defined, required to classification of embeddings for $2m = 3n + 2 - d$. Together with the Haefliger-Wu (i. e. the deleted product) invariant the new invariant gives complete classification of emeddings in the PL caregory for some cases where the Haefliger-Wu invariant is not complete. An interesting and important specific case of these results is classification of embeddings of non-simply connected 4-manifold $S^1 \times S^3$ into R^7 (in the PL category). All the representatives of isotopy classes of embeddings of $S^1 \times S^3$ into R^7 are explicitly constructed. Also it is presented an almost complete classification results for embeddings of $S^p \times S^{2l-1}$ into R^{3l+p}.

In 2006 I delivered lecture courses 'Differential geometry' (II semester, http:// dfgm.math.msu.su/ files/skopenkov/ DIFGEOM.ps), 'Algebraic topology' and 'Charactersitic classes' (II and I semesters, http:// dfgm.math.msu.su/ people/skopenkov/ obstruct2.ps), 'Lie groups and algebras' (II semester) and taught seminar courses 'Differential geometry' (II and I semesters, http:// dfgm.math.msu.su/ files/skopenkov/ DIFGEOM.ps). I taught in the math circles 'Kolmogorov interdiscipline seminar' (II and I semesters, http:// dfgm.math.msu.su/ files/ skopenkov/ kolm.ps) and 'Olympiads and mathematics' (II and I semesters, www.mccme.ru/ circles/oim). These courses except the latter were taught for university students at Faculty of Mechanics and Mathematics of Moscow State University, and the latter was taught for high-school students at Moscow Center for Continuous Mathematical Education. Besides, I taught at various elite summer schools for high-school and university students ('Modern mathematics' summer school, Moscow math olympic schools, Kirov math summer school, summer conference of Tournament of Towns).

25.11.2006 A. Skopenkov