

## Report on the work of A. Skopenkov in 2007 in frame of the Pierre Deligne Stipendium

The following papers were drafted, posted at the internet, submitted, accepted or published in 2007.

[1] A. Skopenkov, Embedding and knotting of manifolds in Euclidean spaces, in: Surveys in Contemporary Mathematics, Ed. N. Young and Y. Choi London Math. Soc. Lect. Notes, 347 (2007) 248–342. <http://arxiv.org/abs/math.GT/0604045>

[2] A. Skopenkov, A characterization of submanifolds by a homogeneity condition, Topol. Appl. 154 (2007) 1894–1897 <http://arxiv.org/abs/math.GT/0606470>

[3] M. Cencelj, D. Repovs and A. Skopenkov, Codimension two PL embeddings of spheres with nonstandard regular neighborhoods, Chinese Annals of Mathematics, Series B, 28:5 (2007) 603–608. <http://arxiv.org/abs/math.GT/0608653>

[4] A. Skopenkov, A new invariant and parametric connected sum of embeddings, Fund. Math. 197 (2007), to appear. <http://arxiv.org/abs/math/0509621> (in 2007 a new version was sent).

[5] A. Skopenkov, A classification of smooth embeddings of 3-manifolds in 6-space, Math. Zeitschrift, to appear. <http://arxiv.org/math.GT/0603429> (in 2007 a new version was sent).

[6] M. Kreck and A. Skopenkov, A classification of smooth embeddings of 4-manifolds in 7-space, submitted. <http://arxiv.org/math.GT/0512594> (in 2007 a new version was sent).

[7] A. Skopenkov, Classification of embeddings below the metastable dimension, preprint, <http://arxiv.org/math.GT/0607422> (in 2007 a new version was sent).

[8] A. Skopenkov, On classification of piecewise-linear embeddings of 4-manifolds in 7-space, draft.

[9] D. Crowley and A. Skopenkov, A classification of smooth embeddings of 4-manifolds in 7-space, II, draft.

The main results are in papers [4–9]. Abstracts of [4–7] are available in the internet; informal description of main results is presented in my report for 2006; in 2007 these papers were significantly rewritten according to remarks by referees.

The new results obtained in 2007 in [5] are Compression Theorems answering the following problem: describe embeddings of 3-manifolds into  $R^6$  are isotopic to an embedding whose image is in  $R^5$  (and the same problems for  $R^5$  replaced by  $R^4$ ). This problem was posed, in particular, by Fomenko, who was motivated by some examples of integrable systems. The proof is based on a new formula for the Kreck invariant.

In [8, 9] we apply methods of [4, 7] and [5, 6], respectively, to classification of embeddings of 4-manifolds in 7-space. In [8] I present an exact sequence involving the set of embeddings of arbitrary  $n$ -manifold into  $R^{2n-1}$  for  $n \geq 4$  (earlier this set was classified known for manifolds with non-empty boundary or simply-connected manifolds). In [9] we generalize the result of [6] to the case when the value of the Boechat-Haefliger-invariant is not primitive.

The paper [1] is a survey paper. The paper [2] contains a very short proof of the smooth homogeneity theorem of D. Repovs, E. V. Scepin and the author. In [3] we proved that any non-locally flat PL embedding of an  $n$ -dimensional sphere in codimension 2 has a regular neighborhood, non-homeomorphic to the standard regular neighborhood  $S^n \times D^2$ .

I wrote reports to papers submitted to research journals and to Möebius Contest (where I worked as a jury member). I wrote a negative report to a habilitation thesis (in the gentle form of an open letter to the author). As a member of Program Committee of

International Conference on Knot Theory in St-Petersburg, September 2007 I suggested to invite this author to the Conference. I participated at the M. M. Postnikov memorial conference (Bedlewo, June 2007) and International Conference on Knot Theory (St-Petersburg, September 2007).

The following pedagogical papers in Russian were posted at the internet, submitted, accepted or published in 2007.

[10] А. Ошемков и А. Скопенков, Олимпиады по геометрии и топологии, Мат. Просвещение, 11 (2007), 131-140. <http://www.mccme.ru/free-books/matprosc.html>

[11] А. Скопенков и А. Телишев, И вновь о критерии Куратовского планарности графов, Мат. Просвещение, 11 (2007), 159–160. <http://www.mccme.ru/free-books/matprosc.html>

[12] А. Скопенков, Алгебраическая топология с элементарной точки зрения, МЦНМО, Москва, в печати, <http://dfgm.math.msu.su/files/skopenkov/obstruct2.ps>

[13] А. Б. Скопенков, Основы дифференциальной геометрии в интересных задачах, МЦНМО, Москва, в печати, <http://dfgm.math.msu.su/files/skopenkov/DIF-GEOM.pdf>

[14] В.И.Богачев, А.М.Райгородский, А.Б.Скопенков и Н.А.Толмачев, Студенческие олимпиады и межкафедральный семинар на мехмате Московского Государственного Университета Мат. Просвещение, 12 (2008), в печати. <http://dfgm.math.msu.su/files/skopenkov/stolymp.pdf>

[15] П. Козлов и А. Скопенков, В поисках утраченной алгебры: в направлении Гаусса (подборка задач), Мат. Просвещение, 12 (2008), в печати [www.mccme.ru/circles/oim/materials/construc.pdf](http://www.mccme.ru/circles/oim/materials/construc.pdf)

[16] А. Скопенков, Исследовательские задачи для школьников, представлено к публикации, [www.mccme.ru/circles/oim/issl.pdf](http://www.mccme.ru/circles/oim/issl.pdf)

[17] А. Скопенков, 13-я проблема Гильберта и базисные вложения, препринт <http://dfgm.math.msu.su/files/skopenkov/hilbert.pdf>

In 2007 I delivered lecture courses 'Differential geometry' (II semester, [13]), 'Algebraic invariant in topological graph theory' (II and I semesters, chapters 0, 1 and 2 of [12]), and taught seminar courses 'Differential geometry' (II and I semesters, [13]). I taught in the math circles 'Kolmogorov interdiscipline seminar' (II and I semesters, <http://dfgm.math.msu.su/files/skopenkov/kolm.ps>) and 'Olympiads and mathematics' (II and I semesters, [www.mccme.ru/circles/oim](http://www.mccme.ru/circles/oim)). These courses except the latter were taught for university students at Faculty of Mechanics and Mathematics of Moscow State University, and the latter was taught for high-school students at Moscow Center for Continuous Mathematical Education. Besides, I taught at various elite summer schools for high-school and university students ('Modern mathematics' summer school, Moscow math olympic schools, Kirov math summer school). I was an advisor of a high-school student's research paper which was accepted to the Russian team at the International Science and Engineering Fair (USA, May 2007).

I organized preparation (in English and Mathematics) of Moscow State University team to the International Competition of Mathematical Students (Bulgaria, Varna, August 2007). I worked as a member of Program Committee of Moscow Mathematical Conference of Schoolpupils and Summer Conference of Tournament of Towns.