Report on the work of A. Skopenkov in 2008
in frame of the Pierre Deligne Stipendium

The following papers were drafted, posted at the internet, submitted, accepted or published in 2008.

[1] A. Skopenkov, Classification of smooth embeddings of 3-manifolds in 6-space, Math. Zeitschrift, 260:3, 2008, 647-672, arxiv:math/0603429.

[2] A. Skopenkov, Embedding and knotting of manifolds in Euclidean spaces, in: Surveys in Contemporary Mathematics, Ed. N. Young and Y. Choi London Math. Soc. Lect. Notes, 347 (2008) 248–342. arxiv:math/0604045.

[3] A. Skopenkov, A classification of smooth embeddings of 4-manifolds in 7-space, submitted, arxiv:math/0512594 (in 2008 a revised version was sent).

[4] D. Crowley and A. Skopenkov, A classification of smooth embeddings of 4-manifolds in 7-space, II, submitted, arxiv:math/0808.1795.

[5] A. Skopenkov, Embeddings of k-connected n-manifolds into R^{2n-k-1}, preprint, arxiv:math/0812.0263.

[6] A. Skopenkov, On classification of embeddings of non-simply-connected 4-manifolds into 7-space, draft (in 2008 a revised version was written).

Abstracts of [1–5] are available in the internet. In this report I do not describe the results of [1,2] presented in my report for 2006 and 2007. The main new results are in papers [3–6].

The papers [3, 4, 6] are on isotopy classification of embeddings of 4-manifolds into 7-dimensional euclidean space R^7 in the *smooth* category. This problem was solved in classical papers only for manifolds with non-empty boundary or in the *piecewise-linear* category for simply-connected manifolds.

In [4] we solve this problem in the *smooth* category for simply-connected manifolds. The proof uses full strength of the Kreck modification of surgery. In the course of the proof we obtain relative diffeomorphism criteria for 7-manifolds with boundary. These criteria are interesting in themselves.

In an earlier paper [3] I solve this problem for a particular case. The proof is much shorter than the proof of [4]. In [3] I also present the construction of attaching invariant and its applications not covered by [4]. (In 2008 the 'attaching invariant' part of [3] was significantly rewritten according to remarks by referee.)

In [6] I present estimations of the set of embeddings of *non-simply-connected* 4-manifold into R^7. The estimations are based on *β-invariant* and *parametric connected sum* of embeddings, both introduced in my recent papers (see reports for 2006 and 2007).

In [5] I prove higher-dimensional versions of 4-dimensional results from [6]. These higher-dimensional versions are easier (so I was able to make final verifications earlier than for the 4-dimensional case) but new and non-trivial.

I participated at the L. S. Pontryagin memorial conference (Moscow, June 2008) and winter school on functional analysis and topology (Voronezh, January 2008).

I wrote reports to papers submitted to research journals, to Möebius Contest (where I worked as a jury member) and to Deligne Contest.

The following books in Russian were posted at the internet and accepted for publication in 2008.

[7] А. Б. Скопенков, Основы дифференциальной геометрии в интересных задачах, МЦНМО, Москва, 2009, http://arxiv.org/abs/0801.1568

[8] А. Скопенков, Алгебраическая топология с элементарной точки зрения, МЦНМО, Москва, в печати, http://arxiv.org/abs/0808.1395

[9] Математика в задачах. Сборник материалов московских выездных математических школ. Под редакцией А. Заславского, Д. Пермякова, А. Скопенкова, М. Скопенкова и А. Шаповалова. www.mccme.ru/circles/oim/mat.htm

The following pedagogical papers in Russian were published in 2008.

[10] А. Скопенков, Размышления об исследовательских задачах для школьников, Мат. Просвещение, 12 (2008), 23–32, www.mccme.ru/circles/oim/issl.pdf

[11] П. Козлов и А. Скопенков, В поисках утраченной алгебры: в направлении Гаусса (подборка задач), Мат. Просвещение, 12 (2008), 127–144, http://arxiv.org/abs/0804.4357

[12] В.И.Богачев, А.М.Райгородский, А.Б.Скопенков и Н.А.Толмачев, Студенческие олимпиады и межкафедральный семинар на мехмате Московского Государственного Университета, Мат. Просвещение, 12 (2008), 205-222, http://dfgm.math.msu.su/files/skopenkov/stolymp.pdf

In 2008 I delivered lecture course 'Algebraic topology from an elementary viewpoint' (II and I semester, [8]), and seminar courses 'Algebraic invariants in topological graph theory' (II semester, chapters 0, 1 and 2 of [8]), 'Kolmogorov interdiscipline seminar' (II and I semesters, http://dfgm.math.msu.su/files/skopenkov/kolm.ps). These courses were taught for students of Faculty of Mechanics and Mathematics of Moscow State University (the lecture course of II semester also for students of the Independent University of Moscow). I was an advisor of a post-graduate student (thesis defence is arranged for 26.12.2008). I took part in preparation of students' mathematical olympiade of Faculty of Mechanics and Mathematics, and in preparation of Moscow State University team to the International Mathematical Competition of university students (Bulgaria, Varna, July 2008).

I taught at math circle 'Olympiads and mathematics' (II and I semesters, www.mccme.ru/circles/oim) for high-school students at Moscow Center for Continuous Mathematical Education. I worked as a member of Program Committee of Moscow Mathematical Conference of Schoolpupils and Summer Conference of Tournament of Towns. Until October, 2008 I was scientific supervisor of internet education system of Moscow Institute of Open Education.

Besides, I taught at various elite summer schools for high-school and university students ('Modern mathematics' summer school, Moscow math olympic schools, Kirov math summer school).

14.12.2008 A. Skopenkov

A list of papers, books and results of A. Skopenkov supported by the Pierre Deligne Stipendium 2006-2008

Research and survey papers.

[1] A. Skopenkov, A new invariant and parametric connected sum of embeddings, Fund. Math. 197 (2007), 253–269. arxiv:math/0509621

[2] A. Skopenkov, A characterization of submanifolds by a homogeneity condition, Topol. Appl. 154 (2007) 1894-1897, http://dx.doi.org/10.1016/j.topol.2007.03.002, arxiv:math/0606470.

[3] M. Cencelj, D. Repovs and A. Skopenkov, Codimension two PL embeddings of spheres with nonstandard regular neighborhoods, Chinese Annals of Mathematics, Series B, 28:5 (2007) 603-608. arxiv:math/0608653

[4] A. Skopenkov, Classification of smooth embeddings of 3-manifolds in 6-space, Math. Zeitschrift, 260:3, 2008, 647-672, arxiv:math/0603429.

[5] A. Skopenkov, Embedding and knotting of manifolds in Euclidean spaces, in: Surveys in Contemporary Mathematics, Ed. N. Young and Y. Choi London Math. Soc. Lect. Notes, 347 (2008) 248–342. arxiv:math/0604045.

[6] A. Skopenkov, A classification of smooth embeddings of 4-manifolds in 7-space, submitted. arxiv:math/0512594

[7] A. Skopenkov, Classification of embeddings below the metastable dimension, submitted, arxiv:math/0607422

[8] D. Crowley and A. Skopenkov, A classification of smooth embeddings of 4-manifolds in 7-space, II, submitted, arxiv:math/0808.1795.

[9] A. Skopenkov, Embeddings of k-connected n-manifolds into R^{2n-k-1}, preprint, arxiv:math/0812.0263.

Main results.

(1) Classification of smooth embeddings of orientable simply-connected 4-manifolds in 7-dimensional euclidean space R^7 (part of the work joint with D. Crowley) [6,8].

(2) Classification of smooth embeddings of orientable 3-manifolds in R^6 [4].

(3) Compression Theorems describing embeddings of 3-manifolds into R^6 which are isotopic to an embedding whose image is in R^5 [4].

(4) Estimations of higher-dimensional isotopy classes of embeddings involving studies of β-invariant and parametric sum of embeddings [1,7,9].

These results are in accordance with research programme. The part of the first result proved in [8] even excels plans suggested in research programme.

Books and pedagogical papers.

[10] A. Skopenkov, Basic Differential Geometry As a Sequence of Interesting Problems, in Russian, MCCME, Moscow, 2009. arXiv:0801.1568

[11] A. Skopenkov, Algebraic topology from an elementary viewpoint, in Russian, MCCME, Moscow, to appear. arXiv:0808.1395

[12] А. Каибханов и А. Скопенков, Примеры трансцендентных чисел, Мат. Просвещение 10 (2006) 176–184, http://www.mccme.ru/free-books/matprosb.html

[13] P. Kozlov and A. Skopenkov, A la recherche de l'algebre perdue: du cote de chez Gauss, Mat. Prosveschenie 12 (2008), 127–144. arXiv:0804.4357

[14] А. Скопенков, Размышления об исследовательских задачах для школьников, Мат. Просвещение, 12 (2008), 23–32, www.mccme.ru/circles/oim/issl.pdf

[15] В.И.Богачев, А.М.Райгородский, А.Б.Скопенков и Н.А.Толмачев, Студенческие олимпиады и межкафедральный семинар на мехмате Московского Государственного Университета, Мат. Просвещение, 12 (2008), 205-222. http://dfgm.math.msu.su/files/skopenkov/stolymp.pdf