The report of Mikhail Bondarko for 2008

1 Scientific activity

The main scientific goal of the project is the study of categories of motives (over a perfect field k). Recall that (Voevodsky's) motives correspond to a certain 'universal cohomology theory for smooth algebraic varieties'. Note that in contrast with 'classical' categories of motives (i.e. Chow, homological and numerical ones) Voevodsky's $DM_{gm}^{eff} \subset DM_{gm}$ are endowed with natural (and highly non-trivial) structures of triangulated categories. One of the main results published is the construction of a close relation of DM_{gm} with $K^b(Chow)$ via a certain exact conservative weight complex functor. I also proved several new results on motives: they concern coniveau spectral sequences, motivic cohomology, and unramified cohomology. One of the main methods was the application of weight structures in triangulaed categories (that I defined in 2007); I developed this technique further and showed that it yields many interesting results for various triangulated categories and cohomological functors.

My paper [1] was published (electronically). The contents of the paper were described in my previous report.

During 2008 I substantially extended my preprint [2] (it is currently 116 pages long). The new parts of the preprint are (new) sections 2.5, 7.4, 7.5, 7.6, 8.2, and 8.6.

Recall that in this preprint a new general formalism for triangulated categories was described. I considered a set of axioms that are (in a certain sense) 'almost dual' to the axioms of t-structures; I called this a weight structure. The easiest example of a weight structure could be described in terms on stupid truncations for the homotopy category of complexes over any additive category. Several properties of weight structures are similar to those of t-structures; yet other ones are quite distinct. A weight structure gives a Postnikov tower of any object which is canonical and functorial up to homotopy. It follows that for any (co)homological functor one obtains a 'weight spectral sequence' whose terms are (co)homology of the corresponding objects of the heart. This spectral sequence is canonical and functorial starting from E_2. One can often obtain t-structures and weight structures from each other by passing to (left and right) *adjacent structures*. In [2] the most important example of the formalism mentioned is Voevodsky's $DM_{gm}^{eff} \subset DM_{gm}$ endowed with a 'Chow' weight structure and adjacent Chow weight and t-structures for DM_{-}^{eff} (note that all of the structures mentioned are new).

In 2008 I proved that the Chow t-structure is closely connected with unramified cohomology; see §7.6 of [2].

I also proved that weight structures could be 'glued' in a manner that is similar to those for t-structures (see §8.2 of [2]). This fact would possibly be used for the construction of the theory of weights for relative motives (Voevodsky's motives over a base that is not a field).

The technique of adjacent structure establishes (and allows to study) a connection of the coniveau filtration on cohomology of motives with the (Voevodsky's) homotopy t-structure on DM_-^{eff}. In particular, torsion motivic cohomology of motives can be expressed in terms certain étale cohomology (here the recently proved Beilinson-Lichtenbaum conjecture is used). Some 'explicit' results on the sort described were proved in §7.4 and §7.5 of [2].

Studying this topic further, I wrote a new preprint [3]. In the case of countable k, I obtain a 'triangulated analogue' of the coniveau spectral sequence. To this end I construct a so-called 'Gersten' weight structure. Since Voevodsky's categories $DM_-^{eff} \supset DM_{gm}^{eff}$ do not contain any motives for infinite field extensions of k (which should generate the heart of this weight structure), I had to construct a certain new triangulated category of *comotives* $\mathfrak{D}$ that contains Voevodsky's DM_{gm}^{eff} as well as (co)motives for arbitrary projective limits of smooth schemes. In this category the motif of a variety was 'decomposed' (in the sense of Postnikov towers) into Tate twists of comotives of its points. The theory of weight structures yields that this result could be generalized to arbitrary objects of DM_{gm}^{eff}. The coniveau spectral sequence obtained (for cohomology of an arbitrary motif) starting from E_2 could be computed in terms of the homotopy t-structure for the category DM_-^{eff} (as in the case of varieties); this generalizes to motives (and extends) the seminal coniveau spectral sequence computations of Bloch and Ogus. I also proved that the (co)motif of a smooth semi-local scheme (or any *primitive* smooth scheme) is a direct summand of the (co)motif of its generic fibre; (co)motives of fields contain twisted (co)motives of their residue fields (for any valuations). Hence similar results hold for any cohomology of (semi-local or primitive) schemes mentioned. I also define a certain exact conservative *weight complex* functor from DM_{gm}^{eff} to the homotopy category of complexes over an (additive) category of *generic motives* (defined by F. Deglise). Recall also that generic motives of Deglise are closely related with cycle modules of M. Rost.

Note also that in order to establish the connection between the (homotopy) t-structure for DM_-^{eff} and the Gersten weight structure for $\mathfrak{D}^s$: $\mathfrak{D}^s \supset DM_{gm}^{eff}$ I developed a general theory of *orthogonal* weight and t-structures; it generalizes the notion of adjacent structures to the case when a pair of (distinct) triangulated categories are equipped with a certain *duality*.

My another topic of research (not directly connected with the main project) was the study of additive Galois modules. Together with A. Dievsky, I published a paper [4]. In the article mixed characteristic wildly ramified local fields extensions were considered. The structure of associated order of such an extension and the question of whether an extension with a given associated order exists was studied. In this paper (in contrast to all preceding literature on associated orders in the 'wild' case) we considered non-abelian extensions. It was proven in the 'self-dual' case that the ring of integers is free over its associated order whenever it could be obtained in a certain (explicity described) way from a certain non-abelian formal group. An infinite series of non-abelian extensions satisfying the conditions described was considered (they are the first non-abelan 'wild' examples of rings of integers free over their associated order). Note that the self-dual case is the only one when the associated order could be a Hopf order;

so it is the only 'wild' case considered by other authors.

The publications (supported by the Balzan foundation) are listed in the end of the report.

2 Pedagogical activity

In 2008 I led student's practice in higher algebra and number theory (in St. Petersburg State University). Besides I actively participated in the composition of a book of problems in Number theory; now it is published and used for teaching students.

Список литературы

[1] Bondarko M.V., Differential graded motives: weight complex, weight filtrations and spectral sequences for realizations; Voevodsky vs. Hanamura// J. of the Inst. of Math. of Jussieu, doi:10.1017/S147474800800011X, **http://journals.cambridge.org/action/displayAbstract?aid=2460176**, 60 pages, see also **http://arxiv.org/abs/math.AG/0601713**

[2] Bondarko M., Weight structures, weight filtrations, weight spectral sequences, and weight complexes for triangulated categories (including motives and spectra), preprint, **http://arxiv.org/abs/0704.4003**

[3] Bondarko M.V., Motivically functorial coniveau spectral sequences for cohomology; direct summands of (co)motives of fields, preprint, **http://arxiv.org/abs/0812.2672**, article password: e6c8s.

[4] Bondarko M.V., Dievsky A.V., Non-abelian associated orders of wildly ramified local field extensions// Zapiski Nauchnyh Seminarov POMI, vol. 356, 5–45, 2008, see **http://www.pdmi.ras.ru/znsl/2008/v356.html**