Pierre Deligne contest.

Shkredov Ilya Dmitrievich, report, 2008.

1. In 2008 I wrote several papers :

- *On sumsets of dissociated sets*, Online Journal of Analytic Combinatorics, 27 pages, submitted for publication,
- *On a result of J. Bourgain* (with S.V. Konyagin), Izvestiya of Russian Academy of Sciences, 33 pages, submitted for publication,
- *On monochromatic solutions of some nonlinear equations in* $\mathbf{Z}/p\mathbf{Z}$, Mat. Zametki, 6 pages, submitted for publication,
- *On some two–dimensional configurations in dense sets*, 97 pages, submitted for publication.

2. About our results.

In the *first paper* we are studying some properties of subsets Q of sums of dissociated sets. (Recall that a set $\Lambda = \{\lambda_1, \dots, \lambda_{|\Lambda|}\}$ is called *dissociated* if the equation $\sum_{i=1}^{|\Lambda|} \varepsilon_i \lambda_i = 0$, where $\varepsilon_i \in \{0, \pm 1\}$ has the only solution $\varepsilon_i = 0$). The exact upper bound for the number of solutions of the following equation

$$q_1 + \dots + q_p = q_{p+1} + \dots + q_{2p}\,, \quad q_i \in Q \tag{1}$$

in groups $\mathbf{F}_2^n$ is found. Using our approach, we easily prove a recent result of J. Bourgain on sets of large exponential sums and obtain a tiny improvement of his theorem. Besides an inverse problem is considered in the article. Let Q be a set belonging a sumset of two dissociated sets such that equation (1) has many solutions. We prove that in the case the large proportion of Q is highly structured.

In his excellent paper on sets without arithmetic progressions of length three J. Bourgain obtained a new upper bound for the density of these sets. One of the crucial moments of the paper was a new result on so–called sets of large exponential sums. *In article "On a theorem of J. Bourgain"* we show that his key result is sharp. We use methods of the first paper and some probabilistic constructions. Also we negatively answer on a natural question of Tom Sanders about Chang's theorem.

Suppose we have an arbitrary finite coloring of natural numbers. Is it true that there are $x, y \in \mathbb{Z}$ such that $x + y$ and xy have the same color? The last (unsolved) question is a well–known problem of Ramsey theory. In the *third paper* we give a positive answer to the question in the group $\mathbb{Z}/p\mathbb{Z}$, where p is a prime number.

A well–known result of Gowers asserts that any set $A \subseteq \{1, \dots, N\}$ of density at least $1/(\log\log N)^{c_k}$, $c_k > 0$ has an arithmetic progression of length k. It is easy to see that the last theorem implies that for an arbitrary finite set $F \subseteq \mathbb{Z}$ there is an affine copy $aF+b$, where $a, b \in \mathbb{Z}$ which belongs to the set A. In the *last article* we consider several two–dimension generalizations of the result for some specific sets F. Let us formulate our main theorem. Let $A \subseteq \{1, \dots, N\}^2$ be a subset of two–dimensional grid of the cardinality $|N|^2/(\log\log\log\log N)^c$, where $c > 0$ is an absolute constant. We prove that A contains a quadruple $\{(x, y), (x, y + r), (x + r, y), (x+2r, y)\}$ and also $\{(x, y), (x+r, y+r), (x+r, y), (x+2r, y)\}$ for some $r \neq 0$. Thus the set F here is $\{(0, 0), (0, 1), (1, 0), (2, 0)\}$ or $\{(0, 0), (1, 1), (1, 0), (2, 0)\}$. Our result is a two–dimensional generalization of Gowers' theorem in the case $k = 4$. Also we obtain a similar statement for subsets A of the group $(\mathbb{Z}/p\mathbb{Z})^n \times (\mathbb{Z}/p\mathbb{Z})^n$, where n is a positive integer, and $p \geq 5$ is a prime number.

3. In the year I took part at the conferences :

- “Uniform distribution” (Marseille, France, January 21–25, 2008),
- “Building bridges” (Budapest, Hungary, August 5–9, 2008),
- “Discrete Rigidity Phenomena in Additive Combinatorics” (Berkeley, USA, November 3–7, 2008).

I gave talks at Minskii number–theoretical Gorodoskoi seminar, Houston mathematical seminar, Additive Combinatorics and Ergodic Theory seminar at MSRI.

4. In our special course "Szemerédi's Theorem and Fourier analysis" we discussed some results of Additive Number Theory and Combinatorial Ergodic Theory. For example we proved classical ergodic H. Furstenberg's proof of Szemerédi theorem and revisited J. Bourgain's result on arithmetic progressions. I read a special course at Moscow Independent Institute about Green and Tao's result on arithmetic progressions in the primes and a course "Ordinary Differential Equations" for second year students at Moscow State University. My student Maxim Makarov studied new Elkin's construction of sets without arithmetic progressions of length three. He gave a talk at our joint seminar with N.G. Moshchevitin "Exponential sums and its applications".