Pierre Deligne contest.

Shkredov Ilya Dmitrievich, report, 2009.

1. In 2009 I wrote several papers :

- *Some applications of W. Rudins inequality to problems of combinatorial number theory*, Uniform Distribution Theory, 15 pages, submitted for publication,
- *On an inverse theorem for* $\mathbf{U}^3(\square)$*-norm*, Modern problems of mathematics and mechanics, Mathematics, Dynamical systems, v. 4, 2, (2009) 55–127.
- *Fourier analysis in combinatorial number theory*, Uspekhi Mat. Nauk, 58 pages, submitted for publication,
- *On Gowers norms of some functions*, 15 pages, Proc. Edinburgh Math. Soc., submitted for publication.

2. About our results.

In the *first paper* we obtain some new applications of a well–known W. Rudin's theorem concerning lacunary series to problems of combinatorial number theory. We prove a sharp generalization of a result of M.–C. Chang on $L_2(\Lambda)$–norm of Fourier coefficients of a set (here Λ is a dissociated set), and obtain a dual version of the theorem. Our main instrument is computing of eigenvalues of some operators.

For a complex function $f : X \to \mathbb{C}$, (X is a finite set, say,) one can define its Gowers norm $\|f\|_{U^k}$ of order k. Gowers inverse theorem says that if $\|f\|_{U^k} \gg 1$ then there is a polynomial $p_{k-1}(x)$ of degree $k-1$ such that $\langle f, p_{k-1}(x) \rangle_P \gg 1$, where P is an arithmetic progression. It is known that Gowers inverse theorem is not sharp for $k \geq 3$ in the sense that its guaranty $|P| \gg |X|^\varepsilon$, $\varepsilon = \varepsilon(\|f\|_{U^k}) > 0$ though a natural upper bound for $|P|$ is $|X|^{1/2}$. In *On an inverse theorem for* $\mathbf{U}^3(\square)$*-norm* we prove a sharp inverse theorem for a two–dimensional variant of Gowers U^3–norm. Besides we discuss some new analogs of Pólya—Vinogradov's inequality of high order which can be obtained using Gowers norms technics.

In *the next* paper we consider some applications of harmonic analysis to combinatorial number theory. Some classical problems of additive combinatorics, coloring problems, high order Fourier analysis, theorems on sets of large exponential sums, results on estimation of exponential sums over subgroups and the connection between combinatorial and analytical number theories are discussed.

A well–known construction of Gowers shows that there are functions with, say, small U^k Gowers norm but large U^{k+1}–norm. The same is true for "skew" variants of Gowers norms, where summation is taken over arbitrary parallelograms. In *the last* paper we consider a family of functions with the property that the smallness of its rectangular norm implies the smallness of rectangular norm for $f(x, x+y)$. Also we study a family of functions $f(x, y)$ having a similar property for higher Gowers norms. The method based on a transference principle for a class of sums over special systems of linear equations.

3. In the year I took part at the conferences :

- "Young Russian Mathematics" (Moscow, Russia, January 12–13, 2009),
- Microsoft Research, consultations (Mountain View, USA, November 10–30, 2009),
- "XXXIV Far East conference mathematical school–seminar dedicated to academician E.V. Zolotov" (Khabarovsk, Russia, June 25–30, 2009),
- "Modern problems of mathematics, mechanics and their applications" (Moscow, Russia, March 30 — April 1, 2009).

4. In our special course "Szemerédi's Theorem and Fourier analysis" we discussed some results of Additive Number Theory. We study classical sum–products theorems in ordered groups, proved Bourgain–Glibichuk–Konyagin and Bourgain–Katz–Tao's theorems on sum–product phenomenon in $\mathbb{Z}/p\mathbb{Z}$ and also uniform distribution of any sufficiently large multiplicative subgroup following a short introduction to the subject of Ben Green. Also we discussed applications Garaev's analytical method to sum–product problems, study Bourgain's expander, Szemerédi—Trotter's theorem and its applications to estimates for some exponential sums in $\mathbb{Z}/p\mathbb{Z}$. Finally, we repeat in the course the classical Burgess–Karacuba's theorem on minimal nonresidual. I read a course "Ordinary Differential Equations" for second year students at Moscow State University. My student Maxim Makarov obtained a result on structure of so–called symmetric sets.