INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Nikolay Konstantinov, Sergey Dorichenko

Why do we conduct the Tournament of Towns? Because we want everything to be well in our house. And our house is the whole world. Not a good idea to use your own words here

N.N.Konstantinov

When a new problem is invented, what should one do? Propose it at an olympiad or throw is out? I think that the same question faces a composer who has created a new melody: he offers it to people if he believes that it will be a gift to them.

N.B.Vasyliev

Nikolai Borisovich Vasyliev was one of the key figures in the jury of the Soviet Union Mathematical Olympiad in the years from 1966 to 1979. A.N. Kolmogorov, who was the jury chairman, formed a team of bright young mathematicians who defined the spirit and style of this competition. But in 1979 government officials disbanded the jury. Then a natural but ambitious idea appeared: to create a new olympiad whose organization would be maximally independent from official Soviet institutions, so it could be controlled by the mathematical community. That is how the Tournament of Towns was created, and N.B. Vasyliev took an active part in this work. He served as the chairman of the Central Jury of the Tournament of Towns till his death in 1998: he selected problems, and was the author of many of them. He also used problems sent by the readers of "Kvant" (Quantum) journal for its problem section. Thanks to N.B. Vasyliev, the Tournament of Towns retained the scientific style of the Soviet Union Mathematical Olympiad while avoiding some of its organizational shortcomings.

We dedicate this article to Nikolai Vasyliev.

WHAT IS THE TOURNAMENT OF TOWNS?

The Tournament of Towns is a worldwide problem solving competition in mathematics for high school students. Its scale is illustrated by the list of participating towns, see the end of this text. Each participating town has a place where students come to solve the problems and write down their solutions; then solutions are graded locally and the best of them are sent for central grading.

So what is special about the Tournament of Towns? How is it different from many other mathematical competitions and why is it being held?

GOALS OF THE TOURNAMENT

The Tournament organizers try to find nice (interesting, non-standard) problems that students could enjoy. Solving these problem during the competition (or after the competition), the students see mathematics from a different angle. In Russia, as in other countries, the standard math classes are usually rather boring and could give the wrong impression that math is a set of recipes for solving standard problems. In general, the school curriculum can not keep up with the rapidly evolving modern world that needs more and more creative people.

Of course, math olympiads organizers do the same. However, there is an important difference. Unlike the International Mathematical Olympiad (where a student must first win some subordinate competitions in order to participate), Tournament of Towns is open for everybody. Every year four rounds are held (basic/advanced autumn/spring competitions) in the participating towns. A student may attend all of them, and only his best performance is taken into account when declaring the "winners". We believe this difference is essential. Indeed, in the IMO pyramid the majority of participants become "losers" at some stage, and only small minority even reaches the final stage. It is quite possible that some participants are discouraged by such a selection system instead of enjoying the problem solving process.

WE ARE LOOKING FOR TALENTS!

The performance of a student is somehow determined by the combination of her talents and the level of teaching (training) she received. The first aspect is more important when we are interested in discovering talented young people who will later attend math schools and universities where they will learn a lot anyway. The competition is more like an appetizer than a dessert — therefore our goal is not to check what a person has learned (like a final exam does), but to guess what she can accomplish. The grading system reflects these priorities. Of course it is necessary that the jury member understands the solution, but the students are given a benefit of doubt and missing steps are acceptable if we believe that they could be in student's mind. This approach has side effects: a solution could be marked as correct while the student doesn't actually fully understand it. Also the students should be discouraged from writing their exam papers in the same style!

THE TOURNAMENT OF TOWNS AND THE MOSCOW MATHEMATICAL OLYMPIAD

The advanced version of the spring round of the Tournament is held on the same day as the Moscow Mathematical Olympiad instead of it. There are several reasons for this decision. First, the spring term in Russian schools is overloaded with various olympiads. Second, the Tournament can be considered as the "export version" of the Moscow Mathematical Olympiad (for other cities), so it is natural that the same problem may appear in both. Therefore, the Moscow Olympiad results are considered on equal footing when determining the "winners" of the Tournament.

THE TOURNAMENT OF TOWNS AS A SPORT

What attracts student to the Tournament? Of course, not only the interesting problems, but also the competition. The desire to test skills and to compete with others is natural for (young) people. An athlete cannot expect his results to be kept a secret, and all results (and the problems, of course) are public.

These two aspects (scientific and competitive) sometimes contradict each other. For beginners the conficts are The athletic side of the Tournament is in conflict with its scientific side. For those who just began being interested in mathematics, this conflict will be unnoticeable. During the Tournament the student is given five hours. For a beginner this is more than enough time to demonstrate their skills, but for a more advanced student, who is capable of solving the hardest problems of the Tournament, five hours is not enough, and for them the Tournament becomes a timed competition. This is in conflict with the spirit of science. To compensate this flaw, the results are formed by looking at three of the student's best solutions. The challenge of the Tournament is that solving three of the hardest problems is comparable to winning the International Mathematical Olympiad, so striving for an even higher results in olympiads is no longer necessary.

THE TOURNAMENT OF TOWNS AND PROFESSIONAL SCIENCE

What should a student who reached such a high level aspire to? He no longer needs olympiads, he needs unsolved mathematical problems. As one of the founders of mathematical olympiads in Russia Boris Nikolaevich Delone said: “An olympiad offers five hours, but you need five thousand hours to solve a serious mathematical problem”.

The Tournament of Towns has a stage, where the students work in a format close to that of a professional mathematician. This stage is the Summer Conference. Unfortunately it is held only for a small number of students (70 to 80 people of the 10 000 participants). Students spend a week there solving problems in a free format. Some parts of these problems are unsolved problems.

SUMMER CONFERENCE

The Tournament’s Summer conference are unlike scientific conferences in the usual sense of the word. They do not have plenary lectures, sectional workgroups or even official programs. These conferences are more like informal gatherings to which student are invited along with accompanying teachers. One of the purposes is to give gifted students access to the solution of research problems. That is why the organizers propose very interesting and difficult problem or cycles of problems frequently connected with real mathematical research. Even the presentation of the statements of such a project can take up a whole lecture, and the presentation of projects can take a whole day. We offer each participant one (maximum two) project, which he will research as deeply as possible.

Solving such problems takes a long time and requires considerable intellectual efforts. So the solving process is rather informal. Usually, several days are given for its attempt, which can either be individual or collaborative.

The participant’s achievements in the Tournament is the main criterion for the invitation to the Conference: those who achieve the highest results are invited. Invitations are also sent to the winners of other prestigious competitions such as All-Russian and IMO. Thus some students from cities and towns where the Tournament is not held can also come to the Conference.

The group leaders are usually the teachers who organise the Tournament in their own cities. Many of them take part in work of the jury of the Conference. The compositions of the jury is not predetermined.

All the participants of the Conference can enjoy sufficient rest, intensive creative work and interesting contacts.

THE JURY DOES NOT ASSIGN PLACES

There is no formal competition during the conference. The Jury simply takes note of what problems are solved, and in the diplomas, which are handed out to the participants, there is only the list of his achievements, but no comparison to the other participants. The overall list of achievements is published, and students can judge by themselves whose achievement is higher.

This is close to real life: there is no jury which can say who is better - Galileo or Newton, Bor or Einstein, Gauss or Euler. Such decisions would be, firstly, of no use to anyone, and secondly, anyone who needs it can decide that for himself.

During the Tournament of Towns there are also no places. In the records published at the end of the Tournament, each of the participants who received a Diploma of a winner of the Tournament has only his best results stated.

There is a line (12 points for the 2015/2016 school year) starting from which students receive diplomas, but the maximal results are much higher.

DIPLOMAS AND AWARDS

In addition to diplomas of the winners of the Tournament, handed out by the Central Jury, the local jury in each participating town can give out their own awards based on their own criteria. In Moscow, students who got at least 5 points but less than 11 are awarded by the Moscow Jury.

Five points approximately corresponds to solving one problem (not the easiest one). This has the following meaning: the difference between a student who solved one problem of medium difficulty and a student who solved nothing is a lot greater than the difference between the student who solved one problem and the student who solved five problems. In the first case the difference is qualitative, in the second it is quantitative. Plus you can add to the students who solved one problem during the Tournament, the ones who solved it after the competition (for example, while riding the Moscow underground, where one often hears, problem solving goes especially well).

OUR WISH FOR THE PARTICIPANTS

In conclusion we a have a request for our participants. All around the world, including Moscow, educators are unjustly shifting their focus from teaching to competitions. Olympiads, tournaments, math battles and other events, which were intended as a means to check the mathematical abilities of students, have gone way beyond mathematical learning, which should form these abilities. We advise you to pay more attention to studying and less to competitions. The main things in studying is to work systematically and not rushing. The great Russian mathematician Igor Rostislavovich Shafarevich once wrote that the wonderful trait of Moscow's mathematical circles is that every question is discussed for as long as it is needed, with no rush. Such style of work is in conflict with the busy rhythm of our lives. To achieve this style you only need to select out all the interesting possibilities - the necessary ones, and out of the necessary ones - the most interesting. Don't spend your time on nonsense, although this is not easy. Don't go around with your eyes closed, try to look at the world with your own eyes, do not blindly follow the authors of books and concepts. Good luck!

REGULATIONS OF THE TOURNAMENT

The Tournament is held each year in two rounds — spring and fall. Students and their cities can take part in either round or both, taking local conditions into consideration. If a certain city participates in both rounds, a student in this city has the right of choosing to take part in only one of them. This does not prevent the student from achieving a good result because the student's score for the Tournament is the maximum (and not the sum) of the scores in the two rounds.

Each round has two levels — O-level and A-level. They are scheduled approximately two weeks apart. Here students have the right of choice as well. They may attempt either level or both. The score for the round is the maximal (not the sum) of the scores in the two levels. The questions in the O-level are less complicated and are accessible to beginners. However, students are awarded less points for solving these questions. Nevertheless students can get enough points to win Diplomas if they solve the hardest three problems of O-level. Questions in the A-level are more complicated. The most difficult of them are often solved only by a few participants. A beginner probably has no chance of obtaining any points from these questions. On the other hand, an exceptional student is sometimes awarded two or three times as many points for them as for O-level questions.

Students who exceed a certain minimum score are awarded a Diploma from the Central Jury. A city's score is based on the average score of the city's best N students' score, where N is the city's population divided by one hundred thousand. If a town's population is less than 500,000, N is then taken to be 5, but the town score is multiplied by a handicap factor.

TOWNS WHICH PARTICIPATE IN THE TOURNAMENT

One of the key traits of the Tournament of Towns is the diversity of participating regions. Because of different cultures, school programs, quality of life and much more, local organizers of the Tournament do things in their own way. For example, in many schools across Argentina the basic round serves as a qualifier for the advanced round round, which is held in two cities - Buenos-Aires and Baia-Blanca. In Iran the Tournament is held as a team competition, in Toronto the basic level is preceded by a set of problems from a math circle at the Moscow State University called Malyi Mechmat MGU.

All local jury's of the Tournament can give out their own awards based on their own criteria, independently of the awards from the Central Jury. This diversity has also affected this publication.

Taiwan is not a city, but a whole country (a politically independent part of China), but in the list below Taiwan is mentioned as a city because the Taiwanese organizers asked to count their region as one big city with a population of 22 million. Several cities in Bulgaria and Israel participate in the Tournament. The overall population of participating cities is around 100 million. Every year around 1000 people are given a diploma of a winner of the Tournament, so for the past 37 Tournaments around 37 thousand people were awarded.

Only a small part of the students on Earth can participate in the Tournament. If one takes into account that there are two million illiterate adults, millions of children who cannot attend school, then it can be seen that our event is still too small to make a difference in the overall mathematical culture of mankind.

Nevertheless some students are lucky enough to discover that mathematics is an endless world of the most refined intellectual creation, capable of satisfying the need of a thinking person. It does not matter if they made this discovery at some olympiad, the Tournament of Towns, at a math circle, or any other way. Even if mathematics did not become their profession, it entered their life forever, leaving a mark on their future work, no matter in which area it might be.

LIST OF TOWNS AND RATING

Notations:

Pop/1000	-	Population of the town, divided by 1000
NoP	-	Number of Participants
NoP Fall	-	Number of Participants in the Fall round
NoP S	-	Number of Participants in the Spring round
Dipl	-	Number of Diplomas
Max	-	Maximum of points, received by participants of the town
NoP cal r (n)	-	Number of best participants' results used for rating calculation in fact (normal)
Av	-	Mean score of the best student's results
C	-	Coefficient for towns with population less than 500 thousands people
Rat	-	Rating of the town (for towns sent to the Central Examination papers of 5 or more students)

Town	Location	Pop /1000	NoP	NoP Fall	NoP S	Dipl	Max	NoP cal r (n)	Av	C	Rat	♯
Kurgan	Russian Fed.	325	34	33	28	26	37.5	5 (5)	29.32	1.22	35.77	1
Troy	Michigan, USA	82	12	11	4	8	24	5 (5)	21.6	1.52	32.83	2
Dolgoprudny	Moscow reg, Russian Fed.	94	30	25	14	29	26.67	5 (5)	21.13	1.51	31.91	3
Ul'yanovsk	Russian Fed.	615	19	16	16	18	34.67	6 (6)	29.39	1	29.39	4
Jeju	Rep. of Korea	435	29	15	18	29	30	5 (5)	27.02	1.08	29.18	5
Irvine	California, USA	223	12	10	6	7	30	5 (5)	21.25	1.35	28.69	6
Gwangmyeong	Rep. of Korea	341	14	10	6	13	26.25	5 (5)	23.32	1.2	27.98	7
Yaroslavl	Russian Fed.	559	23	23	3	18	35	5 (5)	25.95	1	25.95	8
Mokpo	Rep. of Korea	247	14	10	7	13	25.33	5 (5)	19.6	1.32	25.87	9
Kirov	Russian Fed.	483	48	32	36	46	25.33	5 (5)	24.87	1.02	25.36	10
Majkop	Adygea, Russian Fed.	144	27	26	14	16	22.5	5 (5)	17.2	1.44	24.77	11
Belgrade	Serbia	1232	31	28	19	26	37.33	12 (12)	24.44	1	24.44	12
Novorossijsk-BC	Krasnodar reg., Russian Fed.	262	7	5	5	7	22	5 (5)	18.7	1.3	24.31	13
Naberezhnye Chelny	Tatarstan, Russian Fed.	524	33	28	20	26	26	5 (5)	24.03	1	24.03	14
Windsor	Canada	5	5	5	3	3	32.5	5 (5)	14.2	1.62	23	15
Novorossijsk-2	Krasnodar reg., Russian Fed.	262	7	0	7	4	25.33	5 (5)	17.6	1.3	22.88	16
Nizhny Tagil	Sverdlovsk reg., Russian Fed.	358	20	17	12	13	23	5 (5)	19.13	1.18	22.58	17
Ulsan	Rep. of Korea	1163	35	22	19	35	31.5	11 (11)	22.21	1	22.21	18
Zaporizhia	Ukraine	768	43	31	32	14	29.33	7 (7)	22.14	1	22.14	19
Novosibirsk	Russian Fed.	1547	27	24	11	23	32	15 (15)	22.03	1	22.03	20
Zagreb	Croatia	792	5	4	5	5	25	5 (7)	21.9	1	21.9	21
Saransk	Mordovija, Russian Fed.	299	17	17	0	5	21.25	5 (5)	17.32	1.25	21.64	22
Vologda	Russian Fed.	306	33	32	11	15	18.75	5 (5)	17.35	1.24	21.51	23
Helm	Poland	72	22	22	0	11	15	5 (5)	14	1.53	21.42	24
Minsk	Belarus	1912	68	49	41	54	29.33	19 (19)	21.25	1	21.25	25
Krasnodar-BC	Russian Fed.	805	22	18	18	21	26	8 (8)	21.12	1	21.12	26
Kragujevac	Serbia	150	23	0	23	5	17	5 (5)	14.33	1.44	20.64	27
Moscow	Russian Fed.	12184	1382	1377	9	368	33.33	121 (121)	20.24	1	20.24	28
Omsk	Russian Fed.	1160	32	24	25	27	26.25	11 (11)	20.17	1	20.17	29
Petropavlovsk-Kamchatsky	Kamchatka, Russian Fed.	181	10	7	6	3	18	5 (5)	14.37	1.4	20.11	30
Chelyabinsk	Russian Fed.	1182	9	8	5	9	29.33	9 (11)	19.95	1	19.95	31
Seongnam	Rep. of Korea	994	9	6	3	7	42	9 (9)	19.78	1	19.78	32
Seosan	Rep. of Korea	163	6	3	3	3	18.75	5 (5)	13.68	1.42	19.43	33
Ufa	Bashkortostan, Russian Fed.	1077	35	24	14	16	30.67	10 (10)	18.33	1	18.33	34

Town	Location	Pop /1000	NoP	NoP Fall	NoP S	Dipl	Max	NoP cal r (n)	Av	C	Rat	♯
Almaty	Kazahstan	1485	32	17	17	32	22.5	14 (14)	17.91	1	17.91	35
Ekaterinburg	Russian Fed.	1396	35	31	17	17	22.67	13 (13)	17.79	1	17.79	36
Pereslavl-Zalessky	Yaroslavl reg., Russian Fed.	40	8	8	0	2	22.67	5 (5)	11.33	1.57	17.79	37
Kuala Lumpur	Malaysia	1809	11	6	10	10	28.75	11 (18)	17.73	1	17.73	38
Changwon	Rep. of Korea	1089	11	5	7	11	29.33	10 (10)	17.47	1	17.47	39
Kostroma	Russian Fed.	271	7	4	3	6	15	5 (5)	13.53	1.29	17.46	40
Daejeon	Rep. of Korea	1535	20	14	6	16	24	15 (15)	17.23	1	17.23	41
Graz	Austria	276	7	6	6	4	20	5 (5)	13.4	1.28	17.15	42
Gwangju	Rep. of Korea	1477	14	13	4	12	29.33	14 (14)	17.11	1	17.11	43
Zhukovsky	Moscow reg., Russian Fed.	107	8	8	0	2	20	5 (5)	11.27	1.49	16.79	44
Petrozavodsk	Karelia, Russian Fed.	268	7	5	4	3	16	5 (5)	12.98	1.29	16.75	45
Novi Sad	Serbia	341	16	11	9	7	16	5 (5)	13.95	1.2	16.74	46
Sochi-BC	Krasnodar reg., Russian Fed.	473	6	3	3	6	17.33	5 (5)	16.17	1.03	16.65	47
Toronto	Canada	2615	29	18	21	20	34.5	26 (26)	16.58	1	16.58	48
Erevan	Armenia	1068	12	12	0	10	23	10 (10)	16.57	1	16.57	49
Kazan	Tatarstan, Russian Fed.	1176	10	0	10	7	28	10 (11)	16.31	1	16.31	50
Elizovo	Kamchatka, Russian Fed.	38	5	3	3	2	14.67	5 (5)	10.28	1.58	16.25	51
Buenos Aires	Argentina	2890	44	31	32	23	28.75	28 (28)	15.8	1	15.8	52
Vitebsk	Belarus	373	15	11	8	4	16.25	5 (5)	13.52	1.16	15.68	53
Saint Petersburg	Russian Fed.	5028	74	74	0	34	30.67	50 (50)	15.68	1	15.68	54
Calgary	Canada	1096	13	12	7	6	28	10 (10)	15.66	1	15.66	55
Perm	Russian Fed.	1013	75	61	25	15	18.75	10 (10)	15.49	1	15.49	56
Charlotte	North Carolina, USA	792	5	5	0	2	34.67	5 (7)	15.27	1	15.27	57
Jeonju	Rep. of Korea	654	6	2	5	5	22	6 (6)	15.14	1	15.14	58
Astana	Kazahstan	828	31	6	27	14	16	8 (8)	15.09	1	15.09	59
Melbourne	Australia	4250	40	30	28	27	30	40 (42)	14.75	1	14.75	60
Perth	Australia	1832	13	9	9	11	22.67	13 (18)	14.68	1	14.68	61
Samara	Russian Fed.	1171	24	20	10	6	20	11 (11)	14.65	1	14.65	62
Suwon	Rep. of Korea	1170	10	4	6	8	22.5	10 (11)	14.47	1	14.47	63
Goyang	Rep. of Korea	1073	8	3	5	5	28.75	8 (10)	14.41	1	14.41	64
Ivanovo	Russian Fed.	409	5	5	0	3	23	5 (5)	12.95	1.11	14.37	65
Kiev	Ukraine	2849	26	10	17	21	34.5	26 (28)	14.27	1	14.27	66
Chita	Russian Fed.	335	23	21	5	2	20	5 (5)	11.35	1.21	13.73	67
Tomsk	Russian Fed.	557	8	4	5	3	21	5 (5)	13.6	1	13.6	68
Malmö	Sweden	309	5	5	0	2	15	5 (5)	10.92	1.24	13.54	69
Seoul	Rep. of Korea	10117	45	28	20	26	33	45 (101)	13.24	1	13.24	70

Town	Location	Pop /1000	NoP	NoP Fall	NoP S	Dipl	Max	NoP cal r (n)	Av	C	Rat	♯
Saratov	Russian Fed.	840	7	3	4	5	17.33	7 (8)	13.08	1	13.08	71
Cheboksary	Chuvashya, Russian Fed.	464	6	6	1	4	13.33	5 (5)	12.27	1.04	12.76	72
Anyang	Rep. of Korea	609	5	3	2	2	18.75	5 (6)	12.4	1	12.4	73
Isfahan	Iran	1583	16	0	16	8	18	15 (15)	12	1	12	74
Pohang	Rep. of Korea	520	5	5	1	2	23	5 (5)	11.85	1	11.85	75
Busan	Rep. of Korea	3525	25	12	15	11	28	25 (35)	11.73	1	11.73	76
Hamburg	Germany	1718	22	11	17	7	20	17 (17)	11.71	1	11.71	77
Bremen	Germany	544	8	0	8	2	14	5 (5)	11.35	1	11.35	78
Gumi	Rep. of Korea	374	6	4	2	1	16.25	5 (5)	9.7	1.16	11.25	79
Daegu	Rep. of Korea	2492	15	15	2	8	18	15 (24)	11.08	1	11.08	80
Cheongju	Rep. of Korea	838	7	1	6	2	18	7 (8)	10.71	1	10.71	81
Yazd	Iran	486	5	0	5	1	13	5 (5)	10.4	1.02	10.61	82
Voronezh	Russian Fed.	1023	62	62	0	2	15	10 (10)	10.5	1	10.5	83
Rostov-on-Don	Russian Fed.	1103	12	4	9	4	17.33	11 (11)	9.7	1	9.7	84
Kharkiv	Ukraine	1449	10	6	8	3	20	10 (14)	9.43	1	9.43	85
Penza	Russian Fed.	521	20	20	0	1	17	5 (5)	8.8	1	8.8	86
Volzhsky	Volgograd reg., Russian Fed.	326	9	9	0	0	11.25	5 (5)	7	1.22	8.54	87
Kropotkin	Krasnodar reg., Russian Fed.	80	9	5	5	0	7.5	5 (5)	5.55	1.52	8.44	88
Panama	Panama	600	6	6	3	1	19	6 (6)	8.04	1	8.04	89
Incheon	Rep. of Korea	2899	15	6	9	5	15	15 (28)	8.04	1	8.04	90
Tabriz	Iran	2383	6	0	6	2	12	6 (23)	8	1	8	91
Tyumen	Russian Fed.	679	12	6	7	0	10.5	6 (6)	7.92	1	7.92	92
Surgut	Khanty-Mansiy AO, Russian Fed.	332	80	80	0	0	5	5 (5)	4.7	1.21	5.69	93
Sovetsk	Kaliningrad reg., Russian Fed.	41	54	54	0	0	7.5	5 (5)	3.33	1.57	5.23	94
Protvino	Moscow reg., Russian Fed.	37	5	5	0	0	1.25	5 (5)	0.85	1.58	1.34	95
Ejsk-BC	Krasnodar reg., Russian Fed.	85	3	2	2	3	16	3 (5)	14.67	1.52	22.29	–
Vancouver	Canada	603	4	3	3	3	13	4 (6)	12.25	1	12.25	–
Yongin	Rep. of Korea	909	4	2	2	4	24	4 (9)	19.73	1	19.73	–
Makhachkala	Dagestan, Russian Fed.	578	4	3	1	4	17.5	4 (5)	15.38	1	15.38	–
Volgograd	Russian Fed.	1018	3	3	0	3	16	3 (10)	14	1	14	–
Lund	Sweden	107	2	2	0	1	16	2 (5)	8.63	1.49	12.85	–
Essentuki	Stavropol reg., Russian Fed.	103	1	1	0	0	0	1 (5)	0	1.5	0	–
Luga	Leningrad reg., Russian Fed.	36	3	0	3	3	18	3 (5)	15.67	1.58	24.75	–

Town	Location	Pop /1000	NoP	NoP Fall	NoP S	Dipl	Max	NoP cal r (n)	Av	C	Rat	♯
Kaliningrad	Russian Fed.	448	1	0	1	0	8	1 (5)	8	1.06	8.48	–
Nizhny Nov-gorod	Russian Fed.	1259	2	0	2	2	17.33	2 (12)	16.66	1	16.66	–
Boroujen	Iran	49	3	0	3	2	12	3 (5)	10.33	1.56	16.12	–
Khanty-Mansijsk	Russian Fed.	90	3	2	2	3	15	3 (5)	13.67	1.51	20.64	–
Berkley	California, USA	112	1	1	0	1	12	1 (5)	12	1.48	17.76	–
Kerman	Iran	573	3	0	3	2	15	3 (5)	11.33	1	11.33	–

Notes:

1. This rating is counted only for towns sent to the Central examination 5 or more papers.

2. In towns of the Krasnodar land with the note (BC) the Tournament was held by the Bernoulli Centre (Krasnodar).

3. The results of Bremen only for the spring round. The results of Bremen in the fall round are included into the results of Hamburg.

4. Every Iranian team is counted in this rating as one team-participant.

HOW TO ENTER THE TOURNAMENT

The Tournament is open to all towns and cities anywhere in the world. If in the city there is either an education organization (university, institute, school, etc.) which would like to organize the Tournament there, or a group of leaders or even a single teacher who can act as a town committee, this is quite sufficient for the city's participation in the Tournament.

CONTACTS to get the problems and perhaps to join the Tournament: prof. Nikolay Konstantinov (President), Sergey Dorichenko (Chairman of the Jury), turnir.gorodov@gmail.com, turgor@mccme.ru

SELECTED PROBLEMS FROM DIFFERENT YEARS OF THE TOURNAMENT

During its existence the Tournament of Towns has presented its participants over a thousand problems. We present here a select few of these problems for a more meaningful introduction to the Tournament. There is no deep meaning in the choice of problems or their ordering. We wanted to demonstrate the variety of themes - what a participant sees, when he begins solving the problems of yet another round. We tried to order them by increase of difficulty (next to the problem is the number of points which was awarded for its complete solution) The format of this publication prevents us from discussing the solutions. We warn you that:

1. In many of them the answer is surprising or even paradoxical.
2. Some problems are very difficult and were solved by few.

TOURNAMENT 17, Fall 1995
Senior questions, O Level

1. [3 points] A square is placed in the plane and a point P is marked in this plane with invisible ink. A certain person can see this point through special glasses. One can draw a straight line and this person will say on which side of the line the point P lies. If P lies on the line, the person says so. What is the minimal numbers of questions one needs to find out if P lies inside the squares or not?

A. Ya. Kanel-Belov

TOURNAMENT 14, Spring 1993
Junior questions, O Level

2. [3 points] Each of two houses A and B is divided into two flats. Several cats and dogs live there. It is known that the fraction of cats in the first flat of A (the ratio between the number of cats and the total number of animals in the flat) is greater than the fraction of cats in the first flat of B, and the fraction of cats in the second flat of A is greater than the fraction of cats in the second flat of B. Is it true that the fraction of cats in house A is greater than the fractions of cats in house B?

A. K. Kovaldji

TOURNAMENT 9, Spring 1988
Senior questions, O Level

2. [Variant for Moscow participants.] A point has been chosen in a space. Is it possible to arrange four balls in the space so that they do not touch either the point or each other, but "hide" the point in that any ray emanating from the point meets one of the balls?

Problem from Leningrad

TOURNAMENT 4, Spring 1983
Junior questions, O Level

1. [8 points] A pedestrian walked for 3.5 hours. In every period of one hour's duration he walked 5 kilometres. Is it true that his average speed was 5 kilometres per hour?

N. N. Konstantinov

TOURNAMENT 10, Fall 1988
Junior questions, O Level

1. [3 points] It is known that the proportion of people with fair hair among people with blue eyes is more than the proportion of people with fair hair among all people. Which is greater, the proportion of people with blue eyes among people with fair hair, or the proportion of people with blue eyes among all people?

Folklore

TOURNAMENT 12, Fall 1990
Junior questions, O Level

1. [4 points] Suppose two positive real numbers are given. Prove that if their sum is less than their product then their sum is greater than four.

N. B. Vasiliev

TOURNAMENT 17, Spring 1996
Junior questions, O Level

3. [4 points] The two tangents to the incircle of a right-angled triangle ABC the are perpendicular to the hypotenuse AB intersect it at points P and Q. Find $\angle PCQ$.

M. A. Evdokimov

TOURNAMENT 8, Spring 1987
Junior questions, O Level

3. [3 points] We are given two three-litre bottles, one containing 1 litre of water and the other containing 1 litre of 2% salt solution. One can pour liquids from one bottle to the other and then mix them to obtain solutions of different concentration. Can one obtain a 1.5% solution of salt in the bottle which originally contained water?

S. V. Fomin

TOURNAMENT 26, Spring 2005
Junior questions, O Level

3. [5 points] M and N are the midpoints of sides BC and AD, respectively, of a square $ABCD$. K is an arbitrary point on the extension of the diagonal AC beyond A. The segment KM intersects the side AB at some point L. Prove that $\angle KNA = \angle LNA$.

A. V. Akopyan

TOURNAMENT 11, Fall 1989
Senior questions, O Level

4. [3 points] The numbers 2^{1989} and 5^{1989} are written out one after the other (in decimal notation). How many digits are written altogether?

G. A. Galperin

TOURNAMENT 7, Spring 1986
Senior questions

3. [4 points] Vectors coincide with the edges of an arbitrary tetrahedron (possibly non-regular). Is it possible for the sum of these six vectors to equal the zero vector?

Problem from Leningrad

TOURNAMENT 4, Spring 1982
Junior questions, O Level

1. [12 points] There are 36 cards in a deck arranged in the sequence spades, clubs, hearts, diamonds, spades, clubs, hearts, diamonds, etc. Somebody took part of this deck off the top, turned it upside down, and cut this part into the remaining part of the deck (i.e. inserted it between two consecutive cards). Then four cards were taken off the top, then another four, etc. Prove that in any of these sets of four cards, all the cards are of different suits

A. Merkov

TOURNAMENT 4, Fall 1982
Senior questions

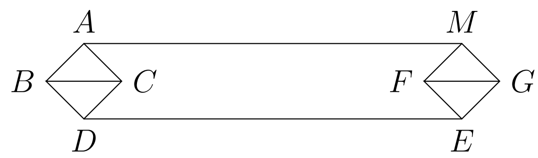
1. [15 points] Prove that for all natural numbers greater than 1

$$[\sqrt{n}] + [\sqrt[3]{n}] + \ldots + [\sqrt[n]{n}] = [\log_2 n] + [\log_3 n] + \ldots + [\log_n n].$$

V. V. Kisil

2. [8 points] Does there exist a poly hedron (not necessary convex) which could have the following complete list of edges?

$$AB, AC, BC, BD, CD, DE, EF, EG, FG, FH, GH, AH$$



Folklore

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TOURNAMENT 24, Fall 2002
Senior questions, A Level

2. [6 points] A cube is cut by a plane so that the cross-section is a pentagon. Prove that the length of one of the sides of the pentagon differs from 1 meter by at least 20 centimeters.

G. A. Galperin

TOURNAMENT 5, Spring 1984
Junior questions, A Level

5. [12 points] The two pairs of consecutive natural numbers $(8, 9)$ and $(288, 289)$ have the following property: in each pair, each number contains each of its prime factors to a power not less than 2. Prove that there are infinitely many such pairs.

A. V. Andjans

TOURNAMENT 6, Spring 1985
Senior questions, A Level

4. [8 points] The convex set F does not cover a semi-circle of radius R. Is it possible that two sets, congruent to F, cover the circle of radius R? What if F is not convex?

N. B. Vasiliev, A. G. Samosvat

5. A square is divided into rectangles. A "chain" is a subset K of the set of these rectangles such that there exists a side of the square which is covered by projections of rectangles of K and such that no point of this side is a projection of two inner points of two different rectangles of K.
(a) [12 points] Prove that every two rectangles in such a division are members of a certain "chain".
(b) [12 points] Solve the similar problem for a cube, divided into rectangular parallelepipeds (in the definition of chain, replace "side" by "edge").

A. I. Golberg, V. A. Gurevich

TOURNAMENT 8, Spring 1987
Senior questions, A Level

3. [5 points] In a certain city only simple (pairwise) exchanges of apartments are allowed (if two families exchange flats, they are not allowed to participate in another exchange on the same day). Prove that any compound exchange may be effected in two days. It is assumed that under any exchange (simple or compound) each family occupies one flat before and after the exchange and the family cannot split up.

A. Shnirelman, N. N. Konstantinov

TOURNAMENT 16, Fall 1994
Senior questions, A Level

3. [4 points] The median AD of triangle ABC intersect its inscribed circle (with center O) at the points X and Y. Find the angle XOY if $AC = AB + AD$.

A. Fedotov

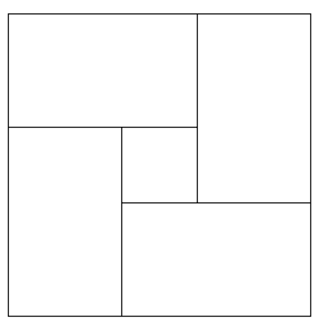
TOURNAMENT 9, Fall 1987
Junior questions, A Level

6. [8 points] There are 2000 apples, contained in several baskets. One can remove baskets and/or remove apples from the baskets. Prove that it is possible to then have an equal number of apples in each of the remaining baskets, with the total number of apples being not less than 100.

A. A. Razborov

TOURNAMENT 6, Spring 1985
Junior questions, A Level

4. [4 points] A square is divided into 5 rectangles in such way that its 4 vertices belong to 4 of the rectangles, whose areas are equal, and the fifth rectangles has no points in common with the side of the squares (see diagram).



Prove that the fifth rectangle is a square.

V. V. Proizvolov

TOURNAMENT 11, Spring 1990
Senior questions, A Level

3. [10 points] A cake is prepared for a dinner party to which only p or q persons will come (p and q are given co-prime integers). Find the minimum number of pieces (not necessary equal) into which the cake must be cut in advance so that the cake may be equally shared between the persons in either case.

D. Fomin, Leningrad

TOURNAMENT 10, Spring 1989
Senior questions, A Level

4. [6 points] A club of 11 people has a committee. At every meeting of the committee a new committee is formed which differs by 1 person from its predecessor (either one new member is included or one member is removed). The committee must always have at least three members and, according to the club rules, the committee membership at any stage must differ from its membership at every previous stage. Is it possible that after some time all possible compositions of the committee will have already occurred?

S. V. Fomin

TOURNAMENT 9, Spring 1988
Senior questions, O Level

4. Pawns are placed on an infinite chess board so that they form an infinite square net (along any row or column containing pawns there is a pawn, three free squares, pawn, three squares, and so on, with only every fourth row and every fourth column containing pawns). Prove that it is not possible for a knight to tour every free square once and only once.

An old problem of A. K .Toplygo

TOURNAMENT 17, Fall 1995
Senior questions, A Level

5. [7 points] Six pine trees grow on the shore of a circular lake. It is known that a treasure is submerged at the mid-point T between the intersection points of the altitudes of two triangles, the vertices of one being at three of the six pines, and the vertices of the second one at the other three pines. At how many points T must one dive to find the treasure?

S. V. Markelov

TOURNAMENT 7, Spring
Senior questions, A Level

6. [8 points] ("Sisyphian labour") There are 1001 steps going up a hill, with rocks on some of them (no more than 1 rock on each step). Sisyphus may pick up any rock and raise it one or more steps up to the nearest empty step. Then his opponent Aid rolls a rock (with an empty step directly below it) down one step. There are 500 rocks, originally located on the first 500 steps. Sisyphus and Aid move rocks in turn, Sisyphus making the first move. His goal is to place a rock on the top step. Can Aid stop him?

S. Yeliseyev

TOURNAMENT 26, Fall 2004
Junior questions, A Level

5. [7 points] Point K belongs to side BC of triangle ABC. Incircles of triangles ABK and ACK touch BC at points M and N respectively. Prove that $BM \cdot CN > KM \cdot KN$.

S. V. Markelov

TOURNAMENT 9, Fall 1987
Senior questions, A Level

7. [8 points] A certain town is represented as an infinite plane, which is divided by straight lines into squares. The lines are streets, while the squares are blocks. Along a certain street there stands a policeman on each 100th intersection. Somewhere in the town there is a bandit, whose position and speed are unknown, but he can move only along the streets. The aim of the police is to see the bandit. Does there exist an algorithm available to the police to enable them to achieve their aim?

A. v. Andjans

TOURNAMENT 26, Fall 2004
Senior questions, A Level

4. [6 points] A circle with the center I is entirely inside of a circle with center O. Consider all possible chords AB of the larger circle which are tangent to the smaller one. Find the locus of the centers of the circles circumscribed about the triangle AIB.

A. A. Zaslavsky

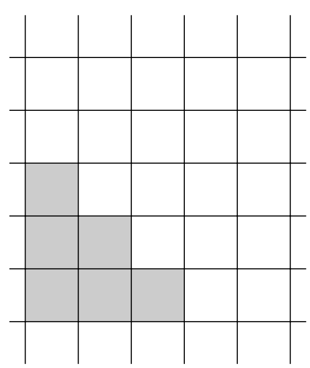
TOURNAMENT 17, Fall 1995
Junior questions, A Level

4. A journalist is looking for a person Z at a meeting of n persons. He has been told that Z knows all the other people at the meeting but none of them knows Z. The journalist may ask any person about any other person: "Do you know that person?" One person can be questioned many times. Al answers are truthful.
(a) [3 points] Can the journalist always find Z by asking less than n questions?
(b) [3 points] What is the minimal number of questions which are needed to find Z?

G. A. Galperin

TOURNAMENT 2, Spring 1981
Senior questions

5. On an infinite "squared" sheet six squares are shaded as in the diagram. On some squares there are pieces. It is possible to transform the positions on the pieces according to the following rule: if the neighbour squares to the right and above a given piece are free, it is possible to remove this piece and put pieces on these free squares.



The goal is to have all the shaded squares free of pieces. Is it possible to reach this goal if
(a) [8 points] In the initial positions there are 6 pieces and they are placed on the 6 shaded squares?
(a) [8 points] In the initial positions there is only one piece, located in the bottom left shaded square?

M. Kontsevich

TOURNAMENT 27, Fall 2005
Senior questions, A Level

5. [7 points] In triangle ABC bisectors AA_1, BB_1 and CC_1 are drawn. Given $\angle A : \angle B : \angle C = 4 : 2 : 1$, prove that $A_1B_1 = A_1C_1$.

S. I. Tokarev

TOURNAMENT 25, Spring 2004
Senior questions, A Level

5. [7 points] The parabola $y = x^2$ intersects a circle at exactly two points A and B. If their tangents at A coincide, must their tangents at B also coincide?

S. V. Markelov

TOURNAMENT 25, Fall 2003
Senior questions, A Level

6. [7 points] An ant crawls on the outer surface of the box in a shape of rectangular parallelepiped. From ant's point of view, the distance between two points on a surface is defined by the length of the shortest path ant need to crawl to reach one point from the other. Is it true that if ant is at vertex then from ant's point of view the opposite vertex be the most distant point on the surface?

S. V. Markelov

LINKS AND BOOKS about Tournament of Towns:
http://www.turgor.ru/en
http://www.math.toronto.edu/oz/turgor

International Mathematics Tournament of Towns. Books 1 — 6. Australian Mathematics Trust. Books 1-3 by P. Taylor, book 4 A. Storozhev and P. Taylor, book 5 by A. Storozhev, book 6 by L. Andy and P. Taylor.

L. Mednikov, A. Shapovalov. Tournament of Towns: world of mathematics in problems. MCCME, 2012. (in Russian)

A. Tolpygo. One thousand probems from the Tournament of Towns. MCCME, 2010. (in Russian)