The Quality of Prediction and Optimal Predictions Based on Two Experts

An. A. Muchnik, A. L. Semenov

Division of Cybernetics at Computing Center of Russian Academy of Sciences

рисунок–дерево

Let the events under consideration satisfy a distribution R. If one of the expert distributions P_1, P_2, P_3, ... is close to the true distribution R, then the distribution Q given by the predictor is close to R as well.

$$W_j(z) = w_j P_j(z) / \sum_i w_i P_i(z)$$

$$Q(za|z) = \sum_j W_j(z) P_j(za|z).$$

Example

Events: a and b.
Types of predictions:

— Deterministic: a with probability 1.

— Stochastic: a with probability 0.99,
b with probability 0.01.

The actual sequence: $a, a, a, \ldots$
Experts: I and II.
For the first 10 000 steps:

— I deterministic,

— II stochastic.

For the next 10 000 steps:

— I stochastic,

— II deterministic.

MDL = I, every time.
Bayes-mixture $\approx$ I, almost every time.
Our predictor = deterministic expert,
almost every time.

Hellinger distance

$$d(r,q) = \sum_{a \in A} \left(\sqrt{r(a)} - \sqrt{q(a)} \right)^2.$$

Distance between two sequences of predictions

$$\sum_{n=0}^{N} d(R(Y_{1:n}a|Y_{1:n}), Q(Y_{1:n}a|Y_{1:n})).$$

True distribution: R.

$$R\left(\left\{Y \,\middle|\, \sum_{n=0}^{N} d(R(Y_{1:n}a|Y_{1:n}), Q(Y_{1:n}a|Y_{1:n})) > s\right\}\right) = \\ = \mathcal{F}_{R,Q}(s).$$

Definition. *Quality of a set* k of predictors is the supremum of all α such that $\exists\, \beta\ \forall s > 0$ there exists a predictor from k such as for any given experts P_j it produces predictions Q with the property $\forall j\ \mathcal{F}_{P_j,Q}(s) < e^{\beta-\alpha s}$.

Theorems for two experts:

I. Quality of set of all predictors is 2. The predictors to achieve this quality can be constructed explicitly (for any given s).

II. Quality of set of all integral predictors is 1. An integral predictor achieving this quality is described.

III. Quality that can be obtained by using Bayes-mixture is $\frac{1}{2}$, by MDL — not greater than $\frac{1}{2}$.

Problem: extend our results for the greater numbers of experts.