Single intermediate degrees in some classical reducibilities

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In his famous work of 1944 Emil Post formulated the problem of existence of an intermediate degree for Turing reducibility and tried to solve it. At the same time he introduced other types of degrees (corresponding to different kinds of reducibilities), defined several classes of sets, in which every element should has an intermediate degree, and showed that these classes are non-void.

In mid-50s the Post's problem was intensively investigated by pupils of Petr Sergeevich Novikov. It was solved by one of them — Al. A. Muchnik and R. M. Friedberg (independently). They constructed a set of *intermediate* degree.

We consider reducibilities introduced by Post (and some other reducibilites) and try to define in an 'explicit' and 'natural' way a single intermediate degree. That kind of intermediate degrees was not known before.

Let us consider computable functions $f : \{0,1\}^* \to \{0,1\}^*$. If $f(x) = y$ we call x a *description* of y *in the mode* f. Let us fix a mode f, then complexity K_f of y is the minimal length of its description. The 'minimal' complexity K is called (*simple* or *Kolmogorov*) *entropy*. For functions f, which domains do not contain words one of which is a proper beginning of another, the corresponding complexity KP is called *prefix entropy*. We define *overgraphs*: $SK = \{(x,n) \mid K(x) < n\}$; $SKP = \{(x,n) \mid KP(x) < n\}$

Known facts about these sets: SK, SKP are recursively enumerable; they are complete with respect to weak truth-table (w-) and, hence Turing (T-) reducibility.

M. Kummer proved that for any entropy K the set SK is complete with respect to truth table (tt-) reducibility.

An. Muchnik found that for some prefix entropies PK the set SPK is complete, for others PK the set SPK is not complete in tt-reducibility; SK and SPK are not complete in bounded Turing (bT-) reducibility. These two results will be published in TCS soon.

Theorems

1. For any two simple entropies their overgraphs are bw-equivalent. The same is true for prefix entropies.

2. Overgraphs of simple entropies and prefix entropies are of intermediate degrees incomparable in bw-.

3. For any two simple entropies their overgraphs are bT-equivalent. The same is true for prefix entropies.

4. For btt- there are non-equivalent sets among overgraphs of SK, the same is true for overgraphs of SPK.

We believe that on the way of investigation of complexities and their relativizations a natural intermediate degree for Turing reducibility will be found.

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