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A vertex operators construction of some not highest or lowest weight modules of the affine Kac–Moody Lie algebras

The aim of the talk is to construct irreducible integrable modules of the non twisted affine Kac–Moody Lie algebras, whose weights are unbounded both from above and from below but which are still realized be means of vertex operators. We shall start with opportune modules of the homogeneous Heisenberg subalgebra $\mathcal{H}$ i.e., a subalgebra isomorphic to the Lie algebra $\mathcal{L}$ with a basis p_i, q_i, c ($i \in \mathbb{N}$) (in this paper $N = \{1, 2, \dots\}$ and commutators:

$$[p_i, q_i] = c\delta_{ij} \quad [p_i, p_j] = 0 \quad [q_i, q_j] = c \quad [c, \mathcal{L}] = 0 \quad i, j \in \mathbb{N}.$$

which are not defined on a Fock space but live on quotients of the representation of the Weyl algebra discovered recently by Billig. The main property of such representations is that there exist a subset I of $\mathbb{N}$ such that the elements of $\mathcal{L}$ q_j $j \in \mathbb{N}$ and p_i $i \in I$ act injectively on it. Nevertheless these modules can be explicitly realized as space of polynomials in an infinite numbers of variables namely $\mathbb{C}[x_1, x_2, \dots, x_{-i_1}, x_{-i_2}, \dots]$ ($i_1, i_2, \dots \in I$) on which the elements of the Heisenberg algebra act as differential or multiplication operators but in a mixed form. The representations of a affine Kac–Moody algebra $\widehat{\mathfrak{g}}$ will be realized on the tensor product of the formal complection of these latter space (i.e., the space of the formal series in the variables x_j, x_{-i} ($j \in \mathbb{N}, i \in I$), and of the group algebra $\mathbb{C}[Q]$ of the root lattice Q of $\widehat{\mathfrak{g}}$ through generalized vertex operators. It can be shown that these latter modules are irreducible and that their weights are unbounded both from below and above or more precisely that they form a dense weight space. Unfortunately it seems that in the most general setting these operators are only defined on the above cited formal completion of the space where the representations of the Heisenberg algebras exist. Nevertheless we shall be able to show that if the subset of elements p_i (or respectively q_j) which act injectively on this module is finite (i.e., the subset denoted above by I) then we can embed our space of polynomials as dense subset in a Hilbert space where (up to the tensor multiplication with $\mathbb{C}[Q]$) all the operators defining the representation of the affine Kac–Moody Lie algebra $\widehat{\mathfrak{g}}$ are well defined (sometimes unbounded) operators.