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Complete commutative algebras on $T^*SO(n)/SO(k_1) \times SO(k_2) \times \cdots \times SO(k_r)$

Mishchenko and Fomenko stated the conjecture that non-commutative integrable systems are integrable in the usual commutative sense by means of integrals that belong to the same functional class as the original non-commutative algebra $\mathcal{F}$ of integrals. If $\mathcal{F}$ is a finite-dimensional Lie algebra the conjecture has been proved recently by Sadetov [4] (see also [5]). In the smooth category, the answer is positive in the infinitely dimensional case as well [2].

The important class of analytical non-commutative integrable systems are geodesic flows of normal metric on homogeneous spaces G/H of compact Lie groups. In this case, the conjecture reduses to the construction of complete commutative algebras within G-invariant functions on $T^*(G/H)$ [2]. For example if (G, H) is a spherical pair, the algebra of G-invariant functions is already commutative.

In many examples, such as Stiefel manifolds, flag manifolds, orbits of the adjoint actions, commutative algebras are obtained (see [2, 3]), but the general problem is still unsolved.

Following [1], by using the shift-argument method, we present the construction of complete commutative algebras for homogeneous spaces $SO(n)/SO(k_1) \times \cdots \times SO(k_r)$ for any choice of $k_1, \ldots, k_r$.

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