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# Lie algebras and singularities theory

In my talk I shall present a number of results about the finite dimensional Lie algebras, which can be naturally associated with the germ of isolated hypersurface singularities (IHS). Recall that for any IHS germ $X := X(f) = \{f = 0\}$, one considers the Lie algebra of derivations $L(X) := O_n/(df)$, where $O_n$ is the algebra of convergent power series in $n$ indeterminates, $f \in O_n$ and $(df)$ is the ideal in $O_n$ generated by all partial derivatives $\partial_i f = \frac{\partial f}{\partial x_i}$. According to S.S-T. Yau, $L(X)$ is a finite dimensional solvable Lie algebra called the Lie algebra of singularity of $X$.

According to Arnold's classification, IHS with modality $\leq 1$ are subdivided into simple IHS ($mod = 0$), parabolic and hyperbolic ones and moreover are 14 exceptional classes. We have investigated Lie algebras of vector field germs for these exceptional singularities, i.e.spaces of derivations of their moduli algebras equipped with the commutator bracket. Namely, explicit bases, and the structure constant presentation in term of these bases have been constructed for derivation Lie algebras of all exceptional singularities.

In the work of Yau and collaborators the corresponding Lie algebras for parabolic singularities have been already computed.

In turns out that in all these cases, the singularities are determined by their Lie algebras, in the sense that all these Lie algebras are pairwise nonisomorphic. Note that for simple singularities this is not the case: Lie algebras of singularities of types $A_6$ and $D_5$ are isomorphic.

Another important property of these Lie algebras is that they are all complete, i.e., they all have trivial centers, and all their derivations are inner.

All these Lie algebras have natural gradings, and Poincare polynomials with respect to these grading have been also calculated for all exceptional IHS.