S. G. Gindikin
Rutgers University
New Brunswick, United States
gindikin@math.rutgers.edu

Complex horospherical transform on real symmetric manifolds

In the end of 50th Gelfand suggested the programm of integral geometry. His plan was to transform the harmonic analysis on homogeneous manifolds with semisimple Lie groups in analytic duality generalizing Radon transform. As in Radon's case the analytic duality must follow to some geometrical duality. The role of projective duality in the general case it must play the horospherical duality. The idea was that it must be an intertwining operator of geometrical nature — the horospherical transform — which differs from harmonic analysis (the decomposition on irreducible representations) on an Abelian Fourier transform. The direct consideration of the decomposition masks the geometrical background.

Gelfand and Graev gave several remarkable examples of the realization of this idea (complex semisimple Lie groups, Riemannian symmetric manifolds of noncompact type) but it was clear that this method does not work in many cases starting of the group $SL(2,\mathbb{R})$: it was not enough of horospheres in these cases. For many years it was unclear, if it is possible to overcome this obstruction. It looks now that there is a quite natural way: if it is not sufficient of real horospheres we can consider the complex ones. We will discuss this idea, starting from the case of compact symmetric spaces and finite dimensional representations where there is an interesting possibility to compare algebraic and analytic approaches.