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Noncommutative del Pezzo surfaces

The hypersurface in a 3-dimensional vector space with an isolated quasi-homogeneous elliptic singularity of type E_r, $r = 6, 7, 8$, has a natural Poisson structure. We show that the family of del Pezzo surfaces of the corresponding type E_r provides a semiuniversal Poisson deformation of that Poisson structure.

We also construct a deformation-quantization of the coordinate ring of such a del Pezzo surface. To this end, we first deform the polynomial algebra $C[x, y, z]$ to a noncommutative algebra with generators x, y, z and the following 3 relations (where $[u, v]_t = uv - tvu$):

$$[x, y]_t = F_1(z), \qquad [y, z]_t = F_2(x), \qquad [z, x]_t = F_3(y)$$

This gives a family of Calabi-Yau algebras $A(F)$ parametrized by a complex number t and a triple $F = (F_1, F_2, F_3)$, of polynomials in one variable of specifically chosen degrees.

Our quantization of the coordinate ring of a del Pezzo surface is provided by noncommutative algebras of the form $A(F)/\langle g\rangle$ where $\langle g\rangle$ stands for the ideal of $A(F)$ generated by a central element g, which generates the center of the algebra $A(F)$ if F is generic enough.