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Elliptic, parabolic and hyperbolic actions of $SL(2,\mathbf{R})$ group

The group $SL(2,\mathbf{R})$ of 2×2 matrices with real entries and the unit determinant acts on the real line by means of linear-fractional (Möbius) transformations:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} : x \mapsto \frac{ax+b}{cx+d}. \tag{1}$$

It is natural to look for an extension of this action into the plane. The most common solution is to consider plane to be a field of complex numbers and define linear-fractional transformations by the same formula (1). This can be naturally identified as an elliptic action.

If we consider the same plane as an algebra of either dual or double numbers [6]*App. C then the same linear-fractional transformations (1) in these non-division rings provide us with corresponding parabolic and hyperbolic actions. This opens a possibility to describe corresponding geometries and analytic function theories in the spirit of Erlangen program.

A suitable modifications allows to extend many fundamental notions of the representation theory (e.g. character, induced representation, etc.) from complex numbers to the cases of non-division rings [4], [3], [5].

An extension of transformations (1) to Banach algebras allows us to review the fundamental notions of functional calculi and spectra [1], [2].

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