D. Luna
Institut Fourier
Grenoble, France
d.luna@wanadoo.fr

# The model wonderful variety

A homogeneous space (under a reductive group $G$) is called a "model homogeneous space", if it is quasi-affine, and if its algebra of regular functions contains every irreducible representation of $G$ exactly once. In [G-Z 84], [G-Z 85] and [A-H-V 98] are given examples of model homogeneous spaces for classical groups and for the simple group of type $E8$.

We will explain the complete classification of model homogeneous for all semi-simple connected groups, obtained in [Lu 07]. For every such group $G$, there exists a unique wonderful variety $M_G$ (called the "model wonderful variety" of $G$) whose orbits parametrize the isomorphisms classes of model homogeneous spaces, in the following way : for every $x \in M_G$, the homogeneous space $G/(G_x)^{\#}$ is a model homogeneous space (where $(G_x)^{\#}$ denotes the kernel of all characters of $G_x$, the isotropy group of $G$ at $x$), and every model homogeneous spaces of $G$ is obtained in this way, up to isomorphism.

We will give details of the (surprisingly varied) structure of $M_G$ for the different simple groups $G$, and will explain the relation with the results of Gel'fand–Zelevisky and Adams–Huang–Vogan mentioned above. We will also mention the relation between our results and recent papers of Brion [Br 06] and [B-CF 06]. Finally, we will explain the ideas of the proof, which uses the combinatorial theory of "spherical systems" and their "augmentations", and talk about the role wonderful varieties play in problems concerning spherical varieties.

## References

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