A. Minchenko
Moscow State University
Moscow, Russia
andrei_msu@mail.ru

# Triads and short $SO_3$-subgroups in compact Lie groups

The report is a remark to one of the results of paper [1].

Let $G$ be a compact Lie group. *Triad* in $G$ is a set of three mutually conjugate involutions (in $G$), product of which is equal to the identity (thus they commute). A triad is said to be *symmetric* if any permutation of its elements is realized in $G$. For example, matrices $\mathrm{diag}(1,-1,-1)$, $\mathrm{diag}(-1,1,-1)$, $\mathrm{diag}(-1,-1,1)$ constitute a symmetric triad in $\mathrm{SO}_3$ (and any other triad is conjugate to it). A subgroup $H \subset G$, isomorphic to $\mathrm{SO}_3$, is said to be *short* $\mathrm{SO}_3$*-subgroup* if the dimension of any irreducible $\mathrm{Ad}\,H$-submodule in $\mathfrak{g}$ does not exceed 5 (i.e. equals 1, 3 or 5).

In what follows $G$ is connected simple compact Lie group without center. In [1] all triads and short $\mathrm{SO}_3$-subgroups in $G$ were found up to conjugacy and on the basis of coincidence of the obtained classifications the following theorem was proved.

**Theorem (Vinberg).** *Any triad in $G$ is obtained in a short $\mathrm{SO}_3$-subgroup, which is uniquely determined up to conjugacy.*

Particularly, all triads in $G$ are symmetric. Vinberg conjectured that there must be some deeper reason for the theorem to be true than just the coincidence of the classifications. The purpose of the report is to show such a reason. The proof is based on the properties of geodesics in symmetric spaces. The fact that $\mathfrak{sl}_2$-subalgebra in $\mathrm{Lie}\,G(\mathbb{C})$ is determined, up to conjugacy, by it's semisimple element (length given a priopi) is used as well.

## References

[1] E. B. Vinberg. Short $\mathrm{SO}_3$ - structures on simple Lie algebras and the associated quasielliptic planes. Amer. Math. Soc. Transl. **213** (2005), 243—270.