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Nilpotent bicone of a reductive Lie algebra

Let $\mathfrak{g}$ be a finite dimensional complex reductive Lie algebra and $\mathfrak{N}_{\mathfrak{g}}$ its nilpotent cone. We denote by $\mathcal{N}_{\mathfrak{g}}$ the subset of elements (x, y) in $\mathfrak{g} \times \mathfrak{g}$ such that the subspace generated by x and y is contained in $\mathfrak{N}_{\mathfrak{g}}$. This subset is called the *nilpotent bicone* of $\mathfrak{g}$. It was first introduced to study the commuting variety of $\mathfrak{g}$. It is naturally endowed with a structure of subscheme of $\mathfrak{g} \times \mathfrak{g}$. We prove, in joint work with J-Y Charbonnel [1], that $\mathcal{N}_{\mathfrak{g}}$, as subscheme of $\mathfrak{g} \times \mathfrak{g}$, is a complete intersection (non reduced) of dimension $3(\mathrm{b}_{\mathfrak{g}} - \mathrm{rk}\mathfrak{g})$, where $\mathrm{b}_{\mathfrak{g}}$ and $\mathrm{rk}\mathfrak{g}$ are respectively the dimension of a Borel subalgebra of $\mathfrak{g}$ and the rank of $\mathfrak{g}$.

In this talk I will present the main steps of our proof (which uses in particular some arguments of motivic integration [2] [5]) and I will give some applications of this result in invariant theory.

References

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