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On Killing vector fields of constant length on homogeneous Riemannian manifolds

The main object of exploration in this talk are *Killing vector fields of constant length* on homogeneous Riemannian manifolds.

A Killing vector field X on a Riemannian manifold (M, g) has constant length if and only if every integral curve of the field X is a geodesic in (M, g). It is known a connections between such fields and one-parameter groups $\gamma(t)$, $t \in \mathbb{R}$, (flows) of *Clifford-Wolf translations* (i.e. isometries moving all points one and the same distance) on a given Riemannian manifold (M, g). First of all, every such flow is generated by a Killing vector field X of constant length on (M, g). If X is a unit Killing vector field on a compact (or on a homogeneous) Riemannian manifold M, then all isometry from the one-parameter group $\gamma(t)$ generated by X $(t \in \mathbb{R})$ with sufficiently small $|t|$ are Clifford-Wolf translations on M. This property is not valid in general for any $t \in \mathbb{R}$. It is remarkable that the flow on symmetric space, generated by a unit Killing vector field, consists of Clifford-Wolf translations [1].

It is a hard problem to obtain a detailed description of Killing vector fields of constant length on arbitrary homogeneous Riemannian manifold. In the second part of the talk we discuss some recent results in this direction. In particular, the following

Theorem. *Let G be a transitive semisimple compact isometry group of a Riemannian manifold M. Suppose that there exists a Killing field $X \in \mathfrak{g}$ of constant length, which is a regular element of $\mathfrak{g}$, where $\mathfrak{g}$ is the Lie algebra of G. Then all $Y \in \mathfrak{g}$ are of constant length, and M is isometric to G supplied with a bi-invariant Riemannian metric.*

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References

[1] *V.N. Berestovskii and Yu.G. Nikonorov*, Killing vector fields of constant length on Riemannian manifolds, Transformation Groups, to appear.