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Chevalley hyperbolic triangle groups in $\mathbb{C}^2$

Let V be a 2-dimensional complex vector space and PV be the complex projective space of V. Fix on V a non-degenerate hermitian form Φ of type $(1,1)$ and define the cone K to be $\{v \in V|\Phi(v,v) < 0\}$. The unitary group $U(\Phi)$ of the form Φ preserves the cone K and the associated projective unitary group $PU(\Phi)$ preserves the 1-dimensional complex hyperbolic space $PK = K/\mathbb{C}^* = \{[v] \in PV : \Phi(v,v) < 0\}$ (as well known this space can be identified with 2-dimensional real hyperbolic space).

Let Γ be a subgroup of $U(\Phi)$. Assume that Γ acts on K as a discrete transformation group. Let $P\Gamma < PU(\Phi)$ be the discrete transformation group of the complex hyperbolic space PK which comes from the complex linear group Γ. We call $P\Gamma$ the "mother group".

Definition. The group $\Gamma < U(\Phi)$ is called the Chevalley hyperbolic group if the following conditions hold:

1) $P\Gamma$ is a discrete subgroup of $PU(\Phi)$ and $PK/P\Gamma$ is compact.

2) $K/\Gamma = \mathbb{C}^2 - \{0\}$

Any Chevalley group is a complex reflection group but this necessary condition is not sufficient. The number of Chevalley hyperbolic groups (up to conjugacy) is finite.

The aim of my talk is to determine all Chevalley hyperbolic groups Γ over the fixed triangle mother-group

$P\Gamma = T(n_1, n_2, n_3) = \langle A, B, C | A^{n_1} = B^{n_2} = C^{n_3} = ABC = 1\rangle$,
$\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} < 1$.