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Free Malcev and alternative superalgebras on one odd generator

(Joint work with I.Shestakov.)

One of important and difficult problems in the theory of non-associative algebras is the construction of effective bases of free algebras. There are not many classes of algebras where such bases are known: free non-associative, free (anti)commutative and free Lie algebras are the most well-known examples besides polynomials and free associative algebras.

Contrary to the classes given above, the free alternative algebra contains non-trivial nilpotent elements and zero divisors [9]; moreover, the free alternative ring has elements of finite additive order [1]. This makes the problem of base more complicated, and it seems natural to consider first some special cases.

For every variety of algebras $\mathcal{V}$, one can consider the corresponding $\mathcal{V}$-Grassmann algebra (see [4]), which is isomorphic as a vector space to the subspace of all skew-symmetric elements of the free $\mathcal{V}$-algebra. Thus, it seems interesting to construct a base for this subspace. Due to [2, 8], the problem is reduced to the free $\mathcal{V}$-superalgebra on one odd generator, which is easer to deal with.

I. Shestakov in [3] constructed a base of the free Malcev superalgebra $\mathcal{M}$ on one odd generator. We continue the study of the superalgebra $\mathcal{M}$ and construct bases of the universal multiplicative envelope $\mathcal{R}(\mathcal{M})$ [4] and the Malcev Poisson superalgebra $\tilde{S}(\mathcal{M})$ related with $\mathcal{M}$ [5]. Using a base of $\tilde{S}(\mathcal{M})$, we eventually construct a base of the free alternative superalgebra $\mathcal{A}$ generated by one odd generator [7]. As a corollary we obtain a base of the alternative Grassmann algebra.

We use the knowledge of the structure of $\mathcal{R}(\mathcal{M})$ to describe the center of $\mathcal{M}$ and to construct a base of the subspace of central skew-symmetric elements in the free Malcev algebra of countable rank. We also describe the nucleus and the center of $\mathcal{A}$ and find a new element of minimal degree in the radical of the free alternative algebra.

The next open problem is speciality of Malcev algebras. It is well known that for any associative algebra A the commutator algebra A^- is a Lie algebra. If A is alternative, then A^- is a Malcev algebra [9].

Recall that Malcev algebra is called *special* if it is isomorphic to a subalgebra of A^- for some alternative algebra A.

We prove that every Malcev superalgebra generated by an odd element is special [6]. As a corollary, we find out two infinite families of non-zero central skew-symmetric elements in the free alternative algebra of countable rank.

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